

**NEHRU COLLEGE OF ENGINEERING AND RESEARCH CENTRE**  
*(Accredited by NAAC, Approved by AICTE New Delhi, Affiliated to APJKTU)*  
**Pampady, Thiruvilwamala(PO), Thrissur(DT), Kerala 680 588**  
**DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGG.**



**COURSE MATERIALS**



**EE 311 ELECTRICAL DRIVES AND CONTROL FOR  
AUTOMATION**

**ABOUT DEPARTMENT**

- ◆ Course offered: B.Tech Electrical and Electronics Engineering
- ◆ Approved by AICTE New Delhi and Accredited by NAAC
- ◆ Affiliated to the University of Dr. AP J Abdul Kalam Technological University.
- ◆

**DEPARTMENT VISION**

To excel in technical education and research in the field of Electrical & Electronics Engineering by imparting innovative engineering theories, concepts and practices to improve the production and utilization of power and energy for the betterment of the Nation.

**DEPARTMENT MISSION**

- To offer quality education in Electrical and Electronics Engineering and prepare the students for professional career and higher studies.
- To create research collaboration with industries for gaining knowledge about real-time problems.

- To prepare students with sound technical knowledge.
  - To make students socially responsible.
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Course code.	Course Name	L-T-P - Credits	Year of Introduction
EE311	ELECTRICAL DRIVES & CONTROL FOR AUTOMATION	3-0-0-3	2016
<b>Prerequisite : Nil</b>			
<b>Course Objectives</b>			
<ol style="list-style-type: none"> <li>To understand the basic concepts of different types of electrical machines and their performance.</li> <li>To know the different methods of starting D.C motors and induction motors.</li> <li>To introduce the controllers for automation</li> </ol>			
<b>Syllabus</b>			
DC Machines, transformers, three phase induction motor, single phase induction motor, stepper motor, controllers for automation.			
<b>Expected outcome .</b>			
The students will be able to			
<ol style="list-style-type: none"> <li>Select a drive for a particular application based on power rating.</li> <li>Select a drive based on mechanical characteristics for a particular drive application.</li> <li>Discuss the controllers used for automation</li> </ol>			
<b>Text Books:</b>			
<ol style="list-style-type: none"> <li>Kothari D. P. and I. J. Nagrath, Electrical Machines, Tata McGraw Hill, 2004.</li> <li>Nagrath .I.J. &amp; Kothari .D.P, Electrical Machines, Tata McGraw-Hill, 1998</li> <li>Richard Crowder, Electrical Drives and Electromechanical systems, Elsevier, 2013</li> <li>Mehta V. K. and R. Mehta, Principles of Electrical and Electronics, S. Chand &amp; Company Ltd., 1996.</li> <li>Theraja B. L. and A. K. Theraja, A Text Book of Electrical Technology, S. Chand &amp; Company Ltd., 2008.</li> <li>Vedam Subrahmaniam, Electric Drives (concepts and applications), Tata McGraw- Hill, 2001</li> </ol>			
<b>References:</b>			
<ol style="list-style-type: none"> <li>H.Partab, Art and Science and Utilisation of electrical energy, Dhanpat Rai and Sons, 1994</li> <li>M. D.Singh, K. B. Khanchandani, Power Electronics, Tata McGraw-Hill, 1998</li> <li>Pillai.S,K A first course on Electric drives, Wiley Eastern Limited, 1998</li> </ol>			
<b>Course Plan</b>			
Module	Contents	Hours	Sem. Exam Marks
I	DC Machines-principle of operation-emf equation-types of excitations. Separately excited, shunt and series excited DC generators, compound generators. General idea of armature reaction, OCC and load characteristics - simple numerical problems.	6	15%
II	Principles of DC motors-torque and speed equations-torque speed characteristics- variations of speed, torque and power with motor current. Applications of dc shunt series and compound motors. Principles of starting, losses and efficiency – load test- simple numerical problems.	6	15%
<b>FIRST INTERNAL EXAMINATION</b>			
III	Transformers – principles of operations – emf equation- vector	7	15%

	diagrams- losses and efficiency – OC and SC tests. Equivalent circuits- efficiency calculations- maximum efficiency – all day efficiency – simple numerical problems. Auto transformers constant voltage transformer- instrument transformers.		
<b>IV</b>	Three phase induction motors- slip ring and squirrel cage types- principles of operation – rotating magnetic field- torque slip characteristics- no load and blocked rotor tests. Circle diagrams- methods of starting – direct online – auto transformer starting	7	15%
<b>SECOND INTERNAL EXAMINATION</b>			
<b>V</b>	Single phase motors- principle of operation of single phase induction motor – split phase motor – capacitor start motor- stepper motor- universal motor Synchronous machines types – emf equation of alternator – regulation of alternator by emf method. Principles of operation of synchronous motors- methods of starting- V curves- synchronous condenser	8	20%
<b>VI</b>	Stepper motors: Principle of operation, multistack variable reluctance motors, single-stack variable reluctance motors, Hybrid stepper motors, Linear stepper motor, comparison, Torque-speed characteristics, control of stepper motors Controllers for automation, servo control, Digital controllers, Advanced control systems, Digital signal processors, motor controllers, Axis controllers, Machine tool controllers, Programmable Logic Controllers	8	20%
<b>END SEMESTER EXAM</b>			

### QUESTION PAPER PATTERN:

**Maximum marks: 100**

**Time: 3 hrs**

The question paper should consist of three parts

**Part A**

There should be 2 questions each from module I and II

Each question carries 10 marks

Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

**Part B**

There should be 2 questions each from module III and IV

Each question carries 10 marks

Students will have to answer any three questions out of 4 (3X10 marks =30 marks)

**Part C**

There should be 3 questions each from module V and VI

Each question carries 10 marks

Students will have to answer any four questions out of 6 (4X10 marks =40 marks)

Note: in all parts each question can have a maximum of four sub questions

## MODULE 1

*DC Machines- principle of operation-emf equation-types of excitations. Separately excited, shunt and series excited DC generators, compound generators. General idea of armature reaction, OCC and load characteristics - simple numerical problems.*

**Introduction**

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator is based on the principle that whenever flux is cut by a conductor, an e.m.f. is induced which will cause a current to flow if the conductor circuit is closed. The direction of induced e.m.f. (and hence current) is given by Fleming's right hand rule. Therefore, the essential components of a generator are:

- (a) a magnetic field
- (b) conductor or a group of conductors
- (c) motion of conductor w.r.t. magnetic field.

**Simple Loop Generator**

Consider a single turn loop ABCD rotating clockwise in a uniform magnetic field with a constant speed as shown in Fig.

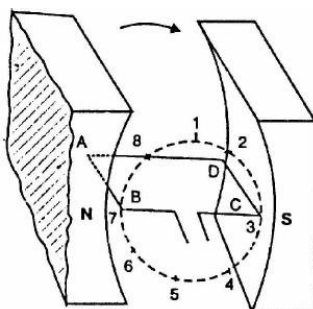


Fig. (1.1)

As the loop rotates, the flux linking the coil sides AB and CD changes continuously. Hence the e.m.f. induced in these coil sides also changes but the e.m.f. induced in one coil side adds to that induced in the other.

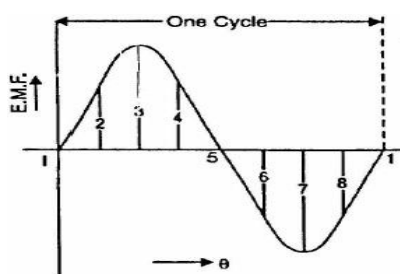


Fig. (1.2)

- (i) When the loop is in position no. 1 [See Fig. 1.1], the generated e.m.f. is zero because the coil sides (AB and CD) are cutting no flux but are moving parallel to it.
- (ii) When the loop is in position no. 2, the coil sides are moving at an angle to the flux and, therefore, a low e.m.f. is generated as indicated by point 2 in Fig. (1.2).
- (iii) When the loop is in position no. 3, the coil sides (AB and CD) are at right angle to the flux and are, therefore, cutting the flux at a maximum rate. Hence at this instant, the generated e.m.f. is maximum as indicated by point 3 in Fig. (1.2).
- (iv) At position 4, the generated e.m.f. is less because the coil sides are cutting the flux at an angle.

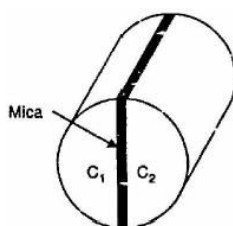
- (v) At position 5, no magnetic lines are cut and hence induced e.m.f. is zero as indicated by point 5 in Fig. (1.2).
- (vi) At position 6, the coil sides move under a pole of opposite polarity and hence the direction of generated e.m.f. is reversed. The maximum e.m.f. in this direction (i.e., reverse direction, See Fig. 1.2) will be when the loop is at position 7 and zero when at position 1. This cycle repeats with each revolution of the coil.

Note that e.m.f. generated in the loop is alternating one. It is because any coil side; say AB has e.m.f. in one direction when under the influence of N-pole and in the other direction when under the influence of S-pole. If a load is connected across the ends of the loop, then alternating current will flow through the load.

The alternating voltage generated in the loop can be converted into direct voltage by a device called commutator. We then have the d.c. generator. In fact, a commutator is a mechanical rectifier.

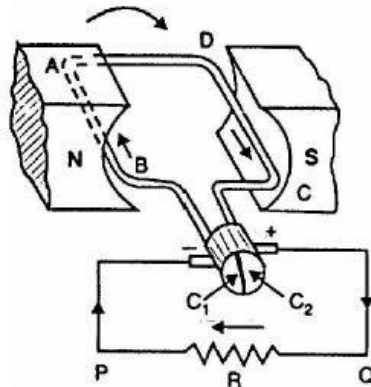
**Commutator**

Connection of the coil side to the external load is reversed at the same instant the current in the coil side reverses, the current through the load will be direct current. This is what a commutator does.



The above shows a commutator having two segments C1 and C2. It consists of a cylindrical metal ring cut into two halves or segments C1 and C2 respectively separated by a thin sheet of mica.

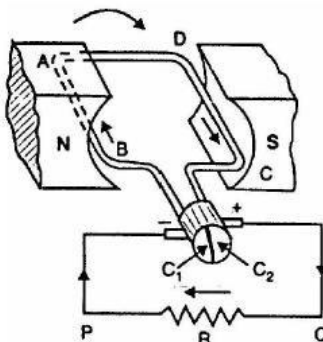
The commutator is mounted on but insulated from the rotor shaft. The ends of coil sides AB and CD are connected to the segments C1 and C2 respectively as shown in Fig.



Two stationary carbon brushes rest on the commutator and lead current to the external load. With this arrangement, the commutator at all times connects the coil side under S-pole to the +ve brush and that under N-pole to the -ve brush.

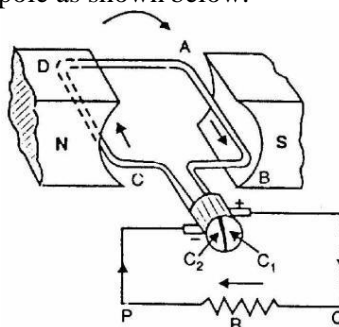
**DC Generator Working**

- (i) In the below Fig., the coil sides AB and CD are under N-pole and S-pole respectively.



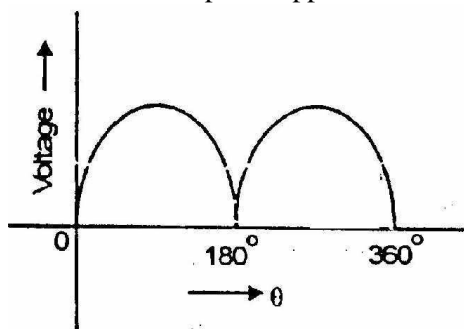
The segment  $C_1$  connects the coil side AB to point P of the load resistance R and the segment  $C_2$  connects the coil side CD to point Q of the load. Also note the direction of current through load. It is from Q to P.

- (ii) After half a revolution of the loop (i.e.,  $180^\circ$  rotation), the coil side AB is under S-pole and the coil side CD under N-pole as shown below.



The currents in the coil sides now flow in the reverse direction but the segments  $C_1$  and  $C_2$  have also moved through  $180^\circ$  i.e., segment  $C_1$  is now in contact with +ve brush and segment  $C_2$  in contact with -ve brush. Note that commutator has reversed the coil connections to the load i.e., coil side AB is now connected to point Q of the load and coil side CD to the point P of the load. Also note the direction of current through the load. It is again from Q to P.

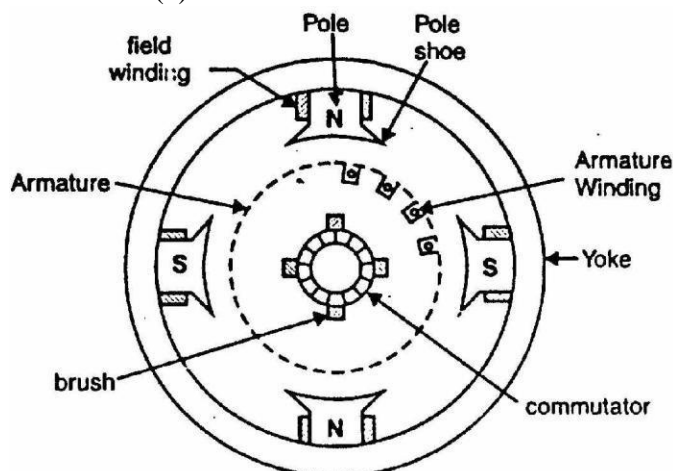
Thus the alternating voltage generated in the loop will appear as direct voltage across the brushes.



The e.m.f. generated in the armature winding of a d.c. generator is alternating one. By the use of commutator we convert the generated alternating e.m.f. into direct voltage. The purpose of brushes is simply to lead current from the rotating loop or winding to the external stationary load.

**Construction of d.c. Generator**

All d.c. machines have five principal components viz., (i) field system (ii) armature core (iii) armature winding (iv) commutator (v) brushes



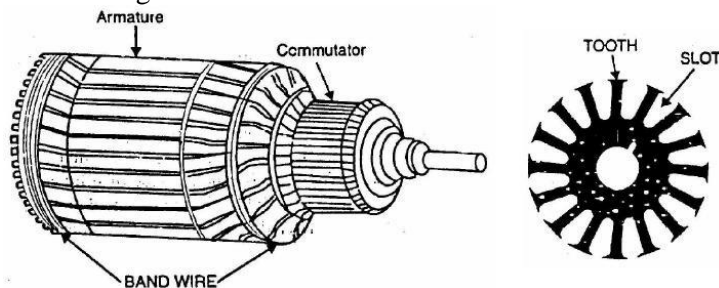
**(i) Field system**

The function of the field system is to produce uniform magnetic field within which the armature rotates. It consists of an even number of salient poles bolted to the inside of circular frame

(generally called yoke). The yoke is usually made of solid cast steel whereas the pole pieces are composed of stacked laminations. Field coils are mounted on the poles and carry the d.c. exciting current. The field coils are connected in such a way that adjacent poles have opposite polarity.

### (ii) Armature core

The armature core is keyed to the machine shaft and rotates between the field poles. It consists of slotted soft-iron laminations (about 0.4 to 0.6 mm thick) that are stacked to form a cylindrical core as shown in Fig.



The laminations are individually coated with a thin insulating film so that they do not come in electrical contact with each other. The purpose of laminating the core is to reduce the eddy current loss. The laminations are slotted to accommodate and provide mechanical security to the armature winding and to give shorter air gap for the flux to cross between the pole face and the armature-teeth.

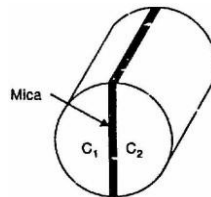
### (iii) Armature winding

The slots of the armature core hold insulated conductors that are connected in a suitable manner. This is known as armature winding. This is the winding in which working e.m.f. is induced.

The armature conductors are connected in series-parallel; the conductors being connected in series so as to increase the voltage and in parallel paths so as to increase the current. The armature winding of a d.c. machine is a closed-circuit winding; the conductors being connected in a symmetrical manner forming a closed loop or series of closed loops.

### (iv) Commutator

A commutator is a mechanical rectifier which converts the alternating voltage generated in the armature winding into direct voltage across the brushes. The commutator is made of copper segments insulated from each other by mica sheets and mounted on the shaft of the machine.

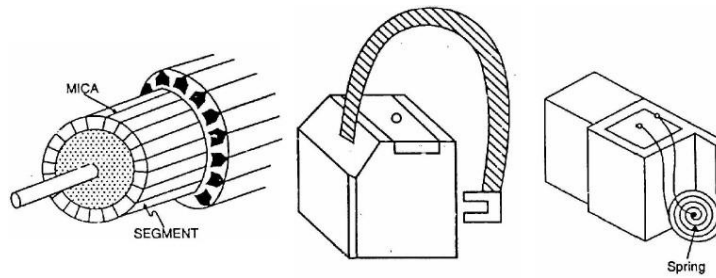


The armature conductors are soldered to the commutator segments in a suitable manner to give rise to the armature winding. Depending upon the manner in which the armature conductors are connected to the commutator segments, there are two types of armature winding in a d.c. machine viz., (a) lap winding (b) wave winding.

### (v) Brushes

The purpose of brushes is to ensure electrical connections between the rotating commutator and stationary external load circuit. The brushes are made of carbon and rest on the commutator. The brush pressure is adjusted by means of adjustable springs.





If the brush pressure is very large, the friction produces heating of the commutator and the brushes. On the other hand, if it is too weak, the imperfect contact with the commutator may produce sparking. Brushes having the same polarity are connected together so that we have two terminals viz., the +ve terminal and the -ve terminal.

### E.M.F. Equation of a D.C. Generator

Let

- $\phi$  = flux/pole in Wb
- $Z$  = total number of armature conductors
- $P$  = number of poles
- $A$  = number of parallel paths = 2 ... for wave winding  
=  $P$  ... for lap winding
- $N$  = speed of armature in r.p.m.
- $E_g$  = e.m.f. of the generator = e.m.f./parallel path

Flux cut by one conductor in one revolution of the armature,  
 $d\phi = P\phi$  webers

Time taken to complete one revolution,  
 $dt = 60/N$  second

e.m.f generated/conductor =  $\frac{d\phi}{dt} = \frac{P\phi}{60/N} = \frac{P\phi N}{60}$  volts

e.m.f. of generator,  
 $E_g =$  e.m.f. per parallel path  
 $=$  (e.m.f/conductor)  $\times$  No. of conductors in series per parallel path  
 $= \frac{P\phi N}{60} \times \frac{Z}{A}$

$\therefore E_g = \frac{P\phi ZN}{60 A}$

where  $A = 2$  for-wave winding  
 $= P$  for lap winding

### Armature Resistance ( $R_a$ )

The resistance offered by the armature circuit is known as armature resistance ( $R_a$ ) and includes:

- (i) resistance of armature winding
- (ii) resistance of brushes

The armature resistance depends upon the construction of machine. Except for small machines, its value is generally less than  $1\Omega$ .

### TYPES OF D.C. GENERATORS

The magnetic field in a d.c. generator is normally produced by electromagnets rather than permanent magnets. Generators are generally classified according to their methods of field excitation. On this basis, d.c. generators are divided into the following two classes:

- (i) Separately excited d.c. generators
- (ii) Self-excited d.c. generators

The behaviour of a d.c. generator on load depends upon the method of field excitation adopted.

#### (i) Separately Excited D.C. Generators

A d.c. generator whose field magnet winding is supplied from an independent external d.c. source (e.g., a battery etc.) is called a separately excited generator.

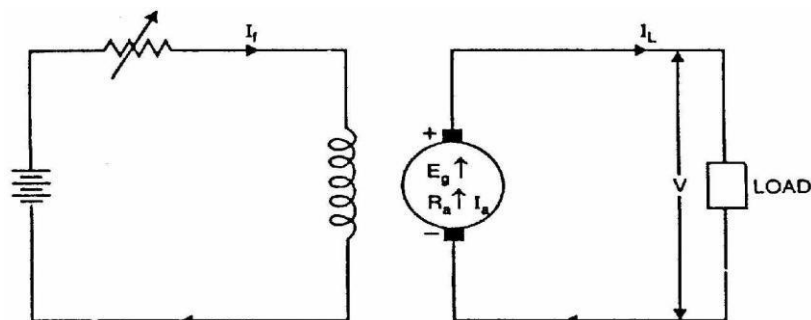


Fig. shows the connections of a separately excited generator.

The voltage output depends upon the speed of rotation of **armature and the field current**

( $E_g = P\phi ZN/60 A$ ). ( $P, Z, 60$  and  $A$  are constants)

**The greater the speed ( $N$ ) and field current ( $\phi$  is directly proportional to  $I_f$ ), greater is the generated e.m.f.**

It may be noted that separately excited d.c. generators are rarely used in practice. The d.c. generators are normally of self-excited type.

Armature current,  $I_a = I_L$

Terminal voltage,  $V = E_g - I_a R_a$

Electric power developed =  $E_g I_a$

Power delivered to load =  $E_g I_a - I_a^2 R_a = I_a (E_g - I_a R_a) = V I_a$

$I_a^2 R_a$  – armature copper loss

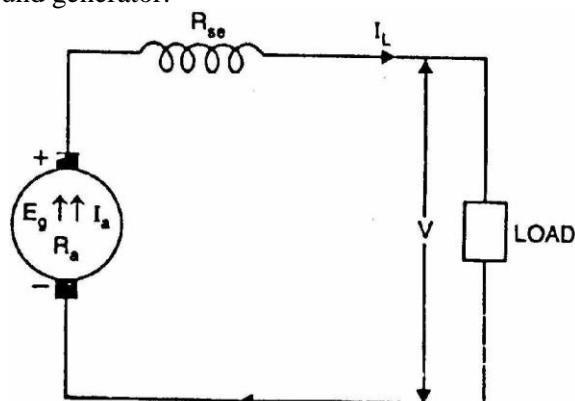
## (ii) Self-Excited D.C. Generators

A d.c. generator whose field magnet winding is supplied current from the output of the generator itself is called a self-excited generator. There are three types of self-excited generators depending upon the manner in which the field winding is connected to the armature, namely;

- (1) Series generator
- (2) Shunt generator
- (3) Compound generator

### (1) Series generator

In a series wound generator, the field winding is connected in series with armature winding so that whole armature current flows through the field winding as well as the load. Fig shows the connections of a series wound generator.



Since the field winding carries the whole of load current, it has a few turns of thick wire having low resistance. Series generators are rarely used except for special purposes e.g., as boosters.

Armature current,  $I_a = I_{se} = I_L = I$  (say)

Terminal voltage,  $V = E_g - I(R_a + R_{se})$

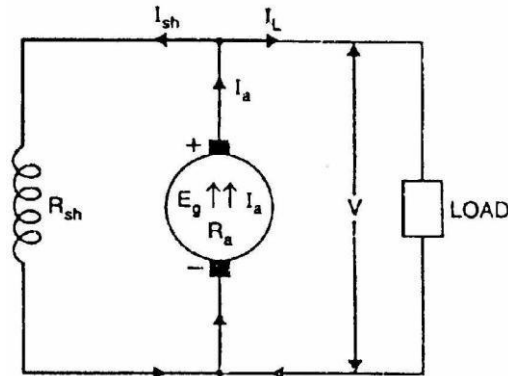
Power developed in armature =  $E_g I_a$

Power delivered to load

$$= E_g I_a - I_a^2 (R_a + R_{se}) = I_a [E_g - I_a (R_a + R_{se})] = V I_a \text{ or } V I_L$$

**(2) Shunt generator**

In a shunt generator, the field winding is connected in parallel with the armature winding so that terminal voltage of the generator is applied across it. The shunt field winding has many turns of fine wire having high resistance. Therefore, only a part of armature current flows through shunt field winding and the rest flows through the load. Fig. shows the connections of a shunt-wound generator.



Shunt field current,  $I_{sh} = V/R_{sh}$

Armature current,  $I_a = I_L + I_{sh}$

Terminal voltage,  $V = E_g - I_a R_a$

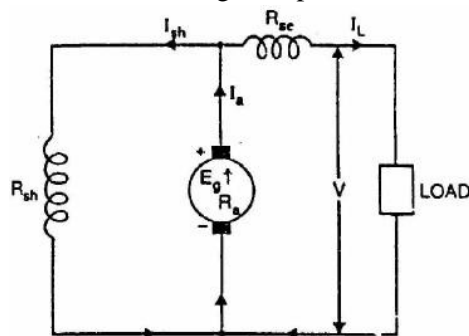
Power developed in armature =  $E_g I_a$

Power delivered to load =  $V I_L$

**(3) Compound generator**

In a compound-wound generator, there are two sets of field windings on each pole—one is in series and the other in parallel with the armature. A compound wound generator may be:

(a) **Short Shunt** in which only shunt field winding is in parallel with the armature winding



Series field current,  $I_{se} = I_L$

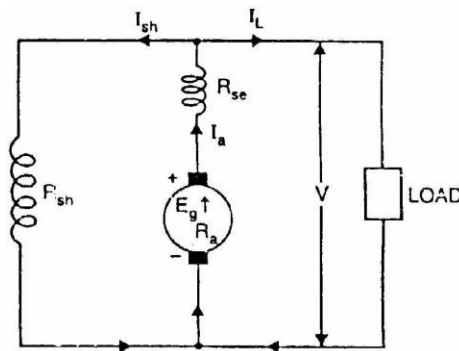
$$\text{Shunt field current, } I_{sh} = \frac{V + I_{se} R_{se}}{R_{sh}}$$

Terminal voltage,  $V = E_g - I_a R_a - I_{se} R_{se}$

Power developed in armature =  $E_g I_a$

Power delivered to load =  $V I_L$

(b) **Long Shunt** in which shunt field winding is in parallel with both series field and armature winding



Series field current,  $I_{se} = I_a = I_L + I_{sh}$

Shunt field current,  $I_{sh} = V/R_{sh}$

Terminal voltage,  $V = E_g - I_a(R_a + R_{se})$

Power developed in armature =  $E_g I_a$

Power delivered to load =  $V I_L$

**Brush Contact Drop**

It is the voltage drop over the brush contact resistance when current flows. Obviously, its value will depend upon the amount of current flowing and the value of contact resistance. This drop is generally small.

**ARMATURE REACTION**

In a d.c. generator, the purpose of field winding is to produce magnetic field (called main flux) whereas the purpose of armature winding is to carry armature current.

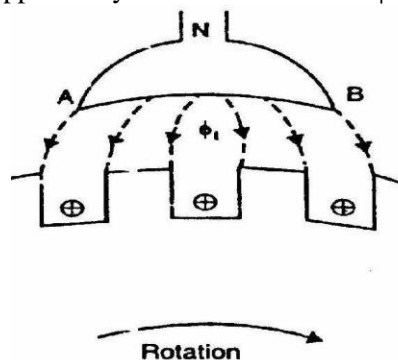
Although the armature winding is not provided for the purpose of producing a magnetic field, nevertheless the current in the armature winding will also produce magnetic flux (called armature flux).

**The armature flux distorts and weakens the main flux posing problems for the proper operation of the d.c. generator. The action of armature flux on the main flux is called armature reaction.**

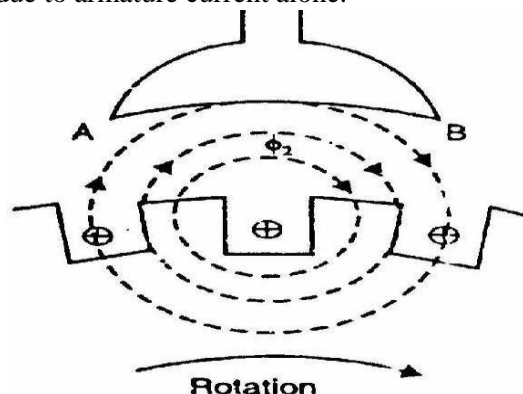
*(The only flux acting in a d.c. machine is that due to the main poles called main flux. However, current flowing through armature conductors also creates a magnetic flux (called armature flux) that distorts and weakens the flux coming from the poles. This distortion and field weakening takes place in both generators and motors. The action of armature flux on the main flux is known as armature reaction.)*

**Explanation**

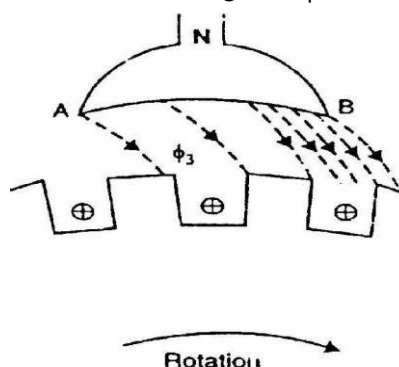
Consider one pole of the generator. When the generator is on no-load, a small current flowing in the armature conductors does not appreciably affect the main flux  $\phi_1$  coming from the pole.



When the generator is loaded, the current flowing through armature conductors sets up flux  $\phi_2$ . The Fig. below shows flux due to armature current alone.



By superimposing  $\phi_1$  and  $\phi_2$ , we obtain the resulting flux  $\phi_3$  as shown in Fig.



From the Fig. it is clear that flux density at the trailing pole tip (point B) is increased while at the leading pole tip (point A) it is decreased. This unequal field distribution produces the following two effects:

1. It demagnetizes or weakens the main flux.
2. It cross-magnetizes or distorts the main flux.

## D.C. GENERATOR CHARACTERISTICS

### 1. Open Circuit Characteristic (O.C.C.)

This curve shows the relation between the generated e.m.f. at no-load ( $E_0$ ) and the field current ( $I_f$ ) at constant speed. It is also known as magnetic characteristic or no-load saturation curve. Its shape is practically the same for all generators whether separately or self-excited. The data for O.C.C. curve are obtained experimentally by operating the generator at no load and constant speed and recording the change in terminal voltage as the field current is varied.

### 2. Internal or Total characteristic ( $E/I_a$ )

This curve shows the relation between the generated e.m.f. on load ( $E$ ) and the armature current ( $I_a$ ).

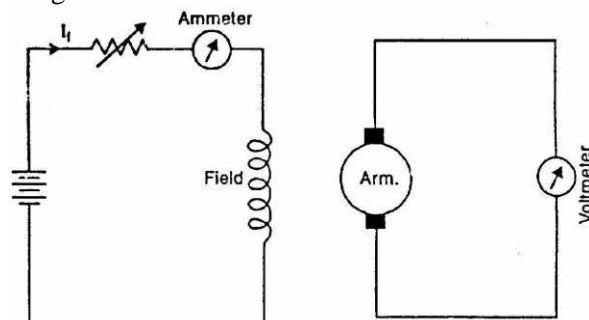
The e.m.f.  $E$  is less than  $E_0$  due to the demagnetizing effect of armature reaction. Therefore, this curve will lie below the open circuit characteristic (O.C.C.). The internal characteristic can be obtained from external characteristic if winding resistances are known because armature reaction effect is included in both characteristics.

### 3. External characteristic ( $V/I_L$ )

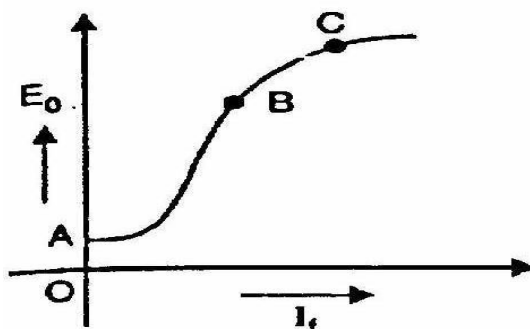
This curve shows the relation between the terminal voltage ( $V$ ) and load current ( $I_L$ ). The terminal voltage  $V$  will be less than  $E$  due to voltage drop in the armature circuit. Therefore, this curve will lie below the internal characteristic. This characteristic is very important in determining the suitability of a generator for a given purpose. It can be obtained by making simultaneous measurements of terminal voltage and load current (with voltmeter and ammeter) of a loaded generator.

### Open Circuit Characteristic of a self excited D.C. Generator

The O.C.C. for a d.c. generator is determined as follows. The field winding of the d.c. generator (series or shunt) is disconnected from the machine and is separately excited from an external d.c. source as shown in Fig.



The generator is run at fixed speed (i.e., normal speed). The field current ( $I_f$ ) is increased from zero in steps and the corresponding values of generated e.m.f. ( $E_0$ ) read off on a voltmeter connected across the armature terminals. On plotting the relation between  $E_0$  and  $I_f$ , we get the open circuit characteristic as shown in Fig.



The following points may be noted from O.C.C.:

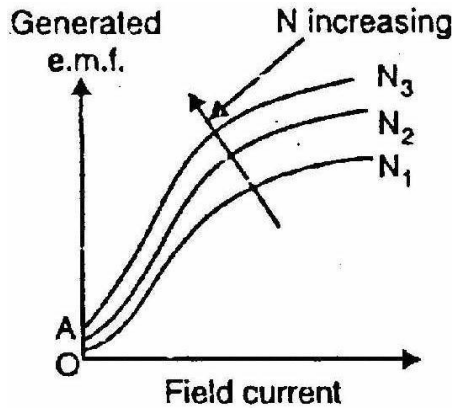
- (i) When the field current is zero, there is some generated e.m.f.  $OA$ . This is due to the residual magnetism in the field poles.
- (ii) Over a fairly wide range of field current (upto point B in the curve), the curve is linear. It is because in this range, reluctance of iron is negligible as compared with that of air gap. The air gap reluctance is constant and hence linear relationship.
- (iii) After point B on the curve, the reluctance of iron also comes into picture. It is because at higher flux densities,  $\mu_r$  for iron decreases and reluctance of iron is no longer negligible. Consequently, the curve deviates from linear relationship.
- (iv) After point C on the curve, the magnetic saturation of poles begins and  $E_0$  tends to level off. The reader may note that the O.C.C. of even self-excited generator is obtained by running it as a separately excited generator.

### Characteristics of a Separately Excited D.C. Generator

The obvious disadvantage of a separately excited d.c. generator is that we require an external d.c. source for excitation. But since the output voltage may be controlled more easily and over a wide range (from zero to a maximum), this type of excitation finds many applications.

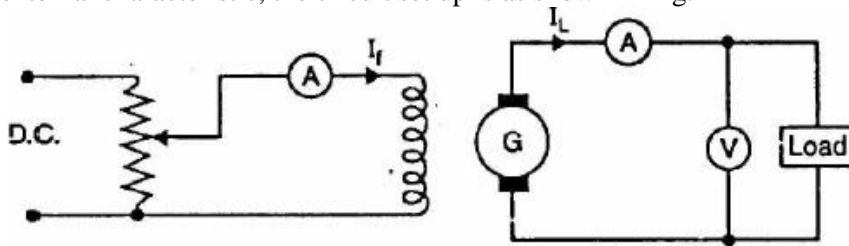
#### (i) Open circuit characteristic.

The O.C.C. of a separately excited generator is determined in a manner described in previous section. Fig. shows the variation of generated e.m.f. on no load with field current for various fixed speeds. Note that if the value of constant speed is increased, the steepness of the curve also increases. When the field current is zero, the residual magnetism in the poles will give rise to the small initial e.m.f. as shown.



**(ii) Internal and External Characteristics**

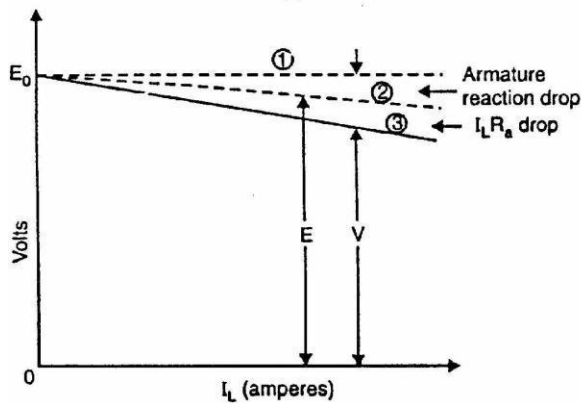
The external characteristic of a separately excited generator is the curve between the terminal voltage (**V**) and the load current  $I_L$  (which is the same as armature current in this case). In order to determine the external characteristic, the circuit set up is as shown in Fig.



As the load current increases, the terminal voltage falls due to two reasons:

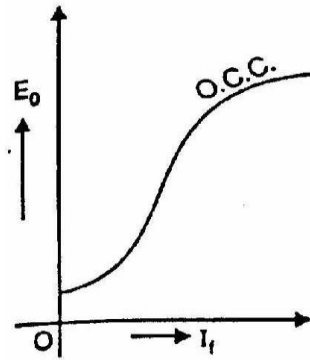
- (a) The armature reaction weakens the main flux so that actual e.m.f. generated  $E$  on load is less than that generated ( $E_0$ ) on no load.
- (b) There is voltage drop across armature resistance ( $= I_L R_a = I_a R_a$ ). Due to these reasons, the external characteristic is a drooping curve [curve 3 in Fig.]. Note that in the absence of armature reaction and armature drop, the generated e.m.f. would have been  $E_0$  (curve 1).

The internal characteristic can be determined from external characteristic by adding  $I_L R_a$  drop to the external characteristic. It is because armature reaction drop is included in the external characteristic. Curve 2 is the internal characteristic of the generator and should obviously lie above the external characteristic.

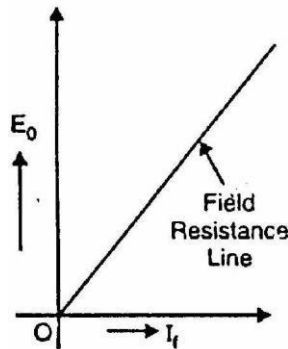


**Voltage Build-Up in a Self-Excited Generator**

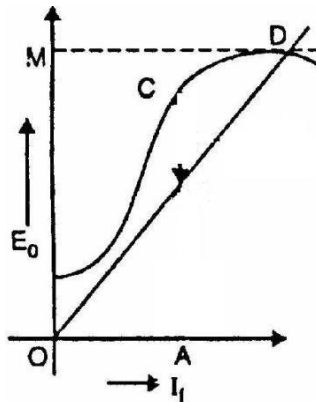
Consider a shunt generator. If the generator is run at a constant speed, some e.m.f. will be generated due to residual magnetism in the main poles. This small e.m.f. circulates a field current which in turn produces additional flux to reinforce the original residual flux (provided field winding connections are correct). This process continues and the generator builds up the normal generated voltage following the O.C.C. shown in Fig.



The field resistance  $R_f$  can be represented by a straight line passing through the origin as shown in Fig.



The voltage build up of the generator is given by the point of intersection of O.C.C. and field resistance line.



Thus in Fig., D is point of intersection of the two curves. Hence the generator will build up a voltage OM.

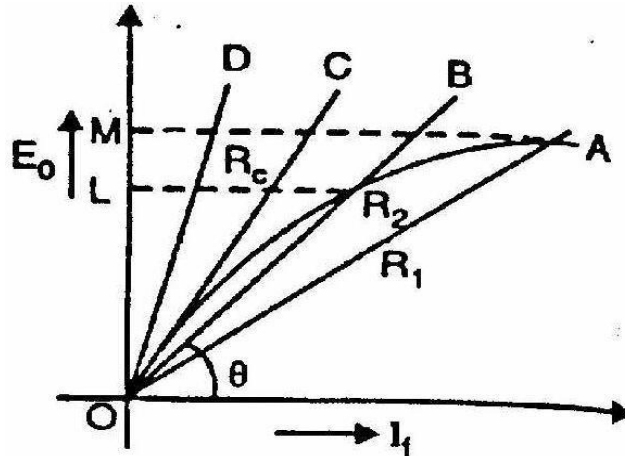
**Critical Field Resistance for a Shunt Generator**

The voltage build up in a shunt generator depends upon field circuit resistance. If the field circuit resistance is  $R_1$  (line OA), then generator will build up a voltage OM as shown in Fig.

If the field circuit resistance is increased to  $R_2$  (line OB), the generator will build up a voltage OL, slightly less than OM.

As the field circuit resistance is increased, the slope of resistance line also increases. When the field resistance line becomes tangent (line OC) to O.C.C., the generator would just excite.





If the field circuit resistance is increased beyond this point (say line OD), the generator will fail to excite. The field circuit resistance represented by line OC (tangent to O.C.C.) is called critical field resistance RC for the shunt generator.

*The maximum field circuit resistance (for a given speed) with which the shunt generator would just excite is known as its critical field resistance.*

**Drawing of OCC at different speed**

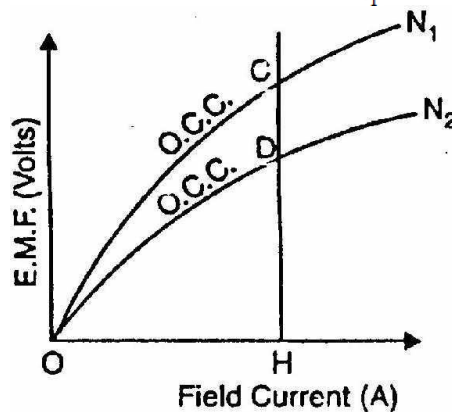
The given O.C.C. of a generator at a constant speed N<sub>1</sub>, then we can easily draw the O.C.C. at any other constant speed N<sub>2</sub>.

Here we are given O.C.C. at a constant speed N<sub>1</sub>. It is desired to find the O.C.C. at constant speed N<sub>2</sub> (it is assumed that N<sub>1</sub> < N<sub>2</sub>).

For constant excitation,  $E \propto N$ .

$$\therefore \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

or  $E_2 = E_1 \times \frac{N_2}{N_1}$



From the above Fig. , for a particular  $I_f = OH$ ,

$$E_1 = HC.$$

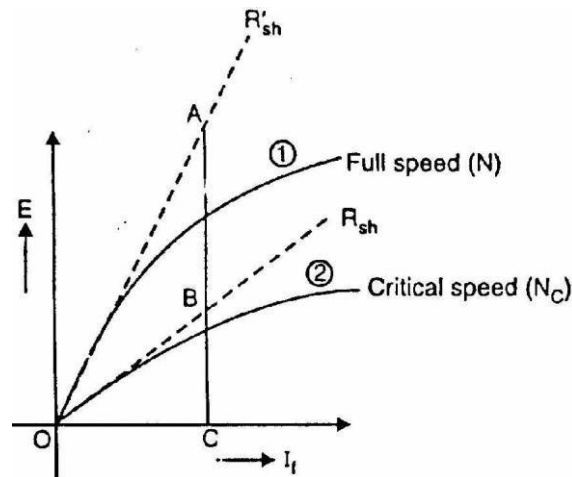
Therefore, the new value of e.m.f. ( $E_2$ ) for the same  $I_f$  but at  $N_2$  is

$$E_2 = HC \times \frac{N_2}{N_1} = HD$$

This locates the point D on the new O.C.C. at  $N_2$ . Similarly, other points can be located taking different values of  $I_f$ . The locus of these points will be the O.C.C. at  $N_2$ .

**Critical Speed (NC)**

The critical speed of a shunt generator is the minimum speed below which it fails to excite. Clearly, it is the speed for which the given shunt field resistance represents the critical resistance.



In Fig. , curve 2 corresponds to critical speed because the shunt field resistance ( $R_{sh}$ ) line is tangential to it.

If the generator runs at full speed  $N$ , the new O.C.C. moves upward and the  $R'_{sh}$  line represents critical resistance for this speed.

$$\therefore \text{Speed} \propto \text{Critical resistance}$$

In order to find critical speed, take any convenient point  $C$  on excitation axis and erect a perpendicular so as to cut  $R_{sh}$  and  $R'_{sh}$  lines at points  $B$  and  $A$  respectively. Then,

$$\frac{BC}{AC} = \frac{N_C}{N}$$

$$\text{or } N_C = N \times \frac{BC}{AC}$$

### Conditions for Voltage Build-Up of a Shunt Generator

The necessary conditions for voltage build-up in a shunt generator are:

- (i) There must be some residual magnetism in generator poles.
- (ii) The connections of the field winding should be such that the field current strengthens the residual magnetism.
- (iii) The resistance of the field circuit should be less than the critical resistance. In other words, the speed of the generator should be higher than the critical speed.

## MODULE 2

*Principles of DC motors-torque and speed equations-torque speed characteristics- variations of speed, torque and power with motor current. Applications of dc shunt series and compound motors. Principles of starting, losses and efficiency – load test- simple numerical problems.*

**D.C. Motor Principle**

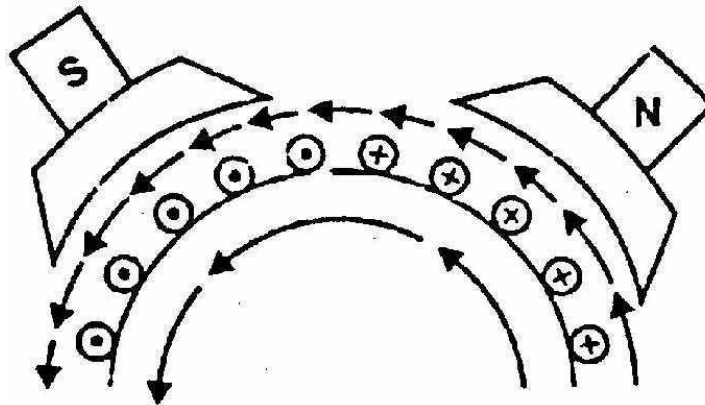
A machine that converts d.c. power into mechanical power is known as a d.c. motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by Fleming's left hand rule and magnitude is given by;

$$F = BI\ell \text{ newtons}$$

Basically, there is no constructional difference between a d.c. motor and a d.c. generator. The same d.c. machine can be run as a generator or motor.

**Working of D.C. Motor**

Consider a part of a multipolar d.c. motor as shown in Fig.



When the terminals of the motor are connected to an external source of d.c. supply:

- (i) the field magnets are excited developing alternate N and S poles;
- (ii) the armature conductors carry currents.

All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction.

Suppose the conductors under N-pole carry currents into the plane of the paper and those under S-pole carry currents out of the plane of the paper as shown in Fig.

Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it. Referring to Fig. and applying Fleming's left hand rule, it is clear that force on each conductor is tending to rotate the armature in anticlockwise direction.

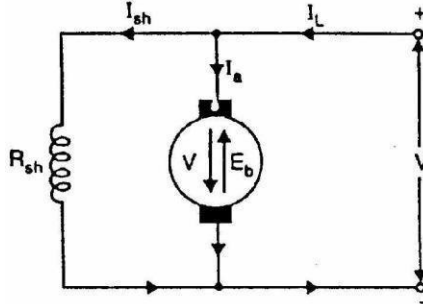
All these forces add together to produce a driving torque which sets the armature rotating.

When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity. Consequently, the direction of force on the conductor remains the same.

**Back or Counter E.M.F.**

When the armature of a d.c. motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence e.m.f. is induced in them as in a generator. The induced e.m.f. acts in opposite direction to the applied voltage  $V$  (Lenz's law) and is known as back or counter e.m.f.  $E_b$ . The back e.m.f.  $E_b (= P \phi ZN/60 A)$  is always less than the applied voltage  $V$ , although this difference is small when the motor is running under normal conditions.

Consider a shunt wound motor shown in Fig.



When d.c. voltage  $V$  is applied across the motor terminals, the field magnets are excited and armature conductors are supplied with current. Therefore, driving torque acts on the armature which begins to rotate. As the armature rotates, back e.m.f.  $E_b$  is induced which opposes the applied voltage  $V$ . The applied voltage  $V$  has to force current through the armature against the back e.m.f.  $E_b$ . The electric work done in overcoming and causing the current to flow against  $E_b$  is converted into mechanical energy developed in the armature. It follows, therefore, that energy conversion in a d.c. motor is only possible due to the production of back e.m.f.  $E_b$ .

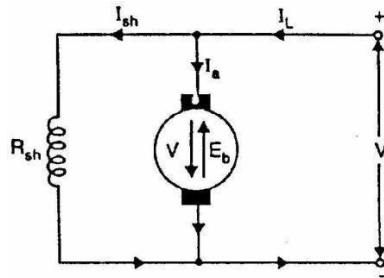
Net voltage across armature circuit =  $V - E_b$   
 If  $R_a$  is the armature circuit resistance, then,

$$I_a = \frac{V - E_b}{R_a}$$

Since  $V$  and  $R_a$  are usually fixed, the value of  $E_b$  will determine the current drawn by the motor. If the speed of the motor is high, then back e.m.f.  $E_b (= P \phi ZN/60 A)$  is large and hence the motor will draw less armature current and vice versa.

**Voltage Equation of D.C. Motor**

Let in a d.c. motor



$V$  = applied voltage

$R_a$  = armature resistance

$E_b$  = back e.m.f.

$I_a$  = armature current

Since back e.m.f.  $E_b$  acts in opposition to the applied voltage  $V$ , the net voltage across the armature circuit is  $V - E_b$ . The armature current  $I_a$  is given by;

$$I_a = \frac{V - E_b}{R_a}$$

$$V = E_b + I_a R_a$$

This is known as voltage equation of the d.c. motor.

**Power Equation**

The above voltage is multiplied by  $I_a$  throughout, we get,

$$VI_a = E_b I_a + I_a^2 R_a$$

This is known as power equation of the d.c. motor.

$VI_a$  = electric power supplied to armature (armature input)

$E_b I_a$  = power developed by armature (armature output)

$I_a^2 R_a$  = electric power wasted in armature (armature Cu loss)

Thus out of the armature input, a small portion (about 5%) is wasted as  $I_a^2 R_a$  and the remaining portion  $E_b I_a$  is converted into mechanical power within the armature.

### Condition for Maximum Power

The mechanical power developed by the motor is  $P_m = E_b I_a$

$$\text{Now } P_m = VI_a - I_a^2 R_a$$

Since,  $V$  and  $R_a$  are fixed, power developed by the motor depends upon armature current. For maximum power,  $dP_m/dI_a$  should be zero.

$$\therefore \frac{dP_m}{dI_a} = V - 2I_a R_a = 0$$

$$\text{or } I_a R_a = \frac{V}{2}$$

$$\text{Now, } V = E_b + I_a R_a = E_b + \frac{V}{2} \quad \left[ \because I_a R_a = \frac{V}{2} \right]$$

$$\therefore E_b = \frac{V}{2}$$

Hence mechanical power developed by the motor is maximum when back e.m.f. is equal to half the applied voltage.

### Limitations

In practice, we never aim at achieving maximum power due to the following reasons:

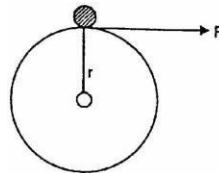
- (i) The armature current under this condition is very large—much excess of rated current of the machine.
- (ii) Half of the input power is wasted in the armature circuit. In fact, if we take into account other losses (iron and mechanical), the efficiency will be well below 50%.

### ARMATURE TORQUE OF D.C. MOTOR

Torque is the turning moment of a force about an axis and is measured by the product of force ( $F$ ) and radius ( $r$ ) at right angle to which the force acts i.e.

$$T = F \times r$$

In a d.c. motor, each conductor is acted upon by a circumferential force  $F$  at a distance  $r$ , the radius of the armature. Therefore, each conductor exerts a torque, tending to rotate the armature.



The sum of the torques due to all armature conductors is known as gross or armature torque ( $T_a$ ).

Let in a d.c. motor

$r$  = average radius of armature in m

$\square$  = effective length of each conductor in m

$Z$  = total number of armature conductors

$A$  = number of parallel paths

$i$  = current in each conductor =  $I_a/A$

$B$  = average flux density in  $\text{Wb/m}^2$

$\phi$  = flux per pole in Wb

$P$  = number of poles

Force on each conductor,  $F = B i \square$  newtons

Torque due to one conductor =  $F \times r$  newton- metre

Total armature torque,  $T_a = Z F r$  newton-metre

$$= Z B i \square r$$

Now  $i = I_a/A$ ,  $B = \phi/a$  where  $a$  is the x-sectional area of flux path per pole at radius  $r$ .

Clearly,  $a = 2\pi r \square /P$ .

$$\begin{aligned} \therefore T_a &= Z \times \left( \frac{\phi}{2} \right) \times \left( \frac{I_a}{A} \right) \times \ell \times r \\ &= Z \times \frac{\phi}{2\pi r \ell / P} \times \frac{I_a}{A} \times \ell \times r = \frac{Z\phi I_a P}{2\pi A} \text{ N - m} \end{aligned}$$

$$\text{or } T_a = 0.159 Z\phi I_a \left( \frac{P}{A} \right) \text{ N - m}$$

Since Z, P and A are fixed for a given machine,

$$\therefore T_a \propto \phi I_a$$

Hence torque in a d.c. motor is directly proportional to flux per pole and armature current.

(i) For a shunt motor, flux  $\phi$  is practically constant.

$$\therefore T_a \propto I_a$$

(ii) For a series motor, flux  $\phi$  is directly proportional to armature current  $I_a$  provided magnetic saturation does not take place.

$$\therefore T_a \propto I_a^2$$

Up to magnetic saturation.

#### Alternative expression for $T_a$

$$E_b = \frac{P\phi ZN}{60A}$$

$$\therefore \frac{P\phi Z}{A} = \frac{60 \times E_b}{N}$$

From Eq.(i), we get the expression of  $T_a$  as:

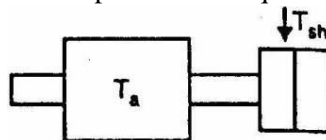
$$T_a = 0.159 \times \left( \frac{60 \times E_b}{N} \right) \times I_a$$

$$\text{or } T_a = 9.55 \times \frac{E_b I_a}{N} \text{ N - m}$$

Note that developed torque or gross torque means armature torque  $T_a$ .

#### Shaft Torque ( $T_{sh}$ )

The torque which is available at the motor shaft for doing useful work is known as shaft torque. It is represented by  $T_{sh}$ . Fig. illustrates the concept of shaft torque.



The total or gross torque  $T_a$  developed in the armature of a motor is not available at the shaft because a part of it is lost in overcoming the iron and frictional losses in the motor. Therefore, shaft torque  $T_{sh}$  is somewhat less than the armature torque  $T_a$ . The difference  $T_a - T_{sh}$  is called lost torque.

$$T_a - T_{sh} = 9.55 \times \frac{\text{Iron and frictional losses}}{N}$$

As stated above, it is the shaft torque  $T_{sh}$  that produces the useful output. If the speed of the motor is  $N$  r.p.m., then,

$$\text{Output in watts} = \frac{2\pi N T_{sh}}{60}$$

$$\text{or } T_{sh} = \frac{\text{Output in watts}}{2\pi N/60} \text{ N - m}$$

$$\text{or } T_{sh} = 9.55 \times \frac{\text{Output in watts}}{N} \text{ N - m} \quad \left( \because \frac{60}{2\pi} = 9.55 \right)$$

### Brake Horse Power (B.H.P.)

The horse power developed by the shaft torque is known as brake horsepower (B.H.P.). If the motor is running at  $N$  r.p.m. and the shaft torque is  $T_{sh}$  newton-metres, then,

$$\begin{aligned} \text{W.D./revolution} &= \text{force} \times \text{distance moved in 1 revolution} \\ &= F \times 2\pi r = 2\pi \times T_{sh} \text{ J} \end{aligned}$$

$$\text{W.D./minute} = 2\pi N T_{sh} \text{ J}$$

$$\text{W.D./sec.} = \frac{2\pi N T_{sh}}{60} \text{ Js}^{-1} \text{ or watts} = \frac{2\pi N T_{sh}}{60 \times 746} \text{ H.P.}$$

$$\therefore \text{Useful output power} = \frac{2\pi N T_{sh}}{60 \times 746} \text{ H.P.}$$

$$\text{or } \text{B.H.P.} = \frac{2\pi N T_{sh}}{60 \times 746}$$

### Speed of a D.C. Motor

$$E_b = V - I_a R_a$$

$$\text{But } E_b = \frac{P\phi ZN}{60 A}$$

$$\therefore \frac{P\phi ZN}{60 A} = V - I_a R_a$$

$$\text{or } N = \frac{(V - I_a R_a) 60 A}{\phi P Z}$$

$$\text{or } N = K \frac{(V - I_a R_a)}{\phi} \quad \text{where } K = \frac{60 A}{P Z}$$

$$\text{But } V - I_a R_a = E_a$$

$$\therefore N = K \frac{E_b}{\phi}$$

$$\text{or } N \propto \frac{E_b}{\phi}$$

Therefore, in a d.c. motor, speed is directly proportional to back e.m.f.  $E_b$  and inversely proportional to flux per pole  $\phi$ .

### Speed Relations

If a d.c. motor has initial values of speed, flux per pole and back e.m.f. as  $N_1$ ,  $\phi_1$  and  $E_{b1}$  respectively and the corresponding final values are  $N_2$ ,  $\phi_2$  and  $E_{b2}$ , then,

$$N_1 \propto \frac{E_{b1}}{\phi_1} \quad \text{and} \quad N_2 \propto \frac{E_{b2}}{\phi_2}$$

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

(i) For a shunt motor, flux practically remains constant so that  $\phi_1 = \phi_2$ .

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

(ii) For a series motor,  $\phi \propto I_a$  prior to saturation.

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

where  $I_{a1}$  = initial armature current  
 $I_{a2}$  = final armature current

### Speed Regulation

The speed regulation of a motor is the change in speed from full-load to no-load and is expressed as a percentage of the speed at full-load i.e.

$$\% \text{ Speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100$$

$$= \frac{N_0 - N}{N} \times 100$$

where  $N_0$  = No - load speed  
 $N$  = Full - load speed

### Torque and Speed of a D.C. Motor

For any motor, the torque and speed are very important factors. When the torque increases, the speed of a motor increases and vice-versa. We have seen that for a d.c. motor;

$$N = K \frac{(V - I_a R_a)}{\phi} = \frac{K E_b}{\phi} \quad \text{(i)}$$

$$T_a \propto \phi I_a \quad \text{(ii)}$$

If the flux decreases, from Eq.(i), the motor speed increases but from Eq.(ii) the motor torque decreases.

This is not possible because the increase in motor speed must be the result of increased torque. Indeed, it is so in this case. When the flux decreases slightly, the armature current increases to a large value. As a result, in spite of the weakened field, the torque is momentarily increased to a high value and will exceed considerably the value corresponding to the load. The surplus torque available causes the motor to accelerate and back e.m.f. ( $E_a = P \phi Z N/60A$ ) to rise. Steady conditions of speed will ultimately be achieved when back e.m.f. has risen to such a value that armature current [ $I_a = (V - E_a)/R_a$ ] develops torque just sufficient to drive the load.

### D.C. MOTOR CHARACTERISTICS

There are three principal types of d.c. motors viz., shunt motors, series motors and compound motors. Both shunt and series types have only one field winding wound on the core of each pole of the motor. The compound type has two separate field windings wound on the core of each pole. The



performance of a d.c. motor can be judged from its characteristic curves known as motor characteristics, following are the three important characteristics of a d.c. motor:

**(i) Torque and Armature current characteristic ( $T_a/I_a$ )**

It is the curve between armature torque  $T_a$  and armature current  $I_a$  of a d.c. motor. It is also known as electrical characteristic of the motor.

**(ii) Speed and armature current characteristic ( $N/I_a$ )**

It is the curve between speed  $N$  and armature current  $I_a$  of a d.c. motor. It is very important characteristic as it is often the deciding factor in the selection of the motor for a particular application.

**(iii) Speed and torque characteristic ( $N/T_a$ )**

It is the curve between speed  $N$  and armature torque  $T_a$  of a d.c. motor. It is also known as mechanical characteristic.

**CHARACTERISTICS OF SHUNT MOTORS**

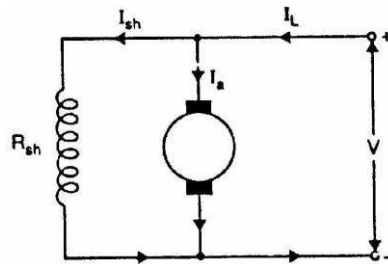


Fig. shows the connections of a d.c. shunt motor. The field current  $I_{sh}$  is constant since the field winding is directly connected to the supply voltage  $V$  which is assumed to be constant. Hence, the flux in a shunt motor is approximately constant.

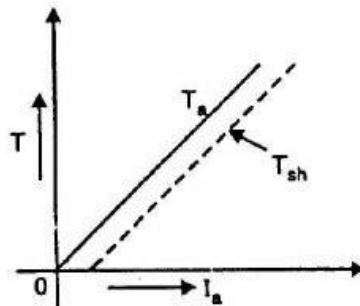
**(i)  $T_a/I_a$  Characteristic.** In a d.c. motor,

$$T_a \propto \phi I_a$$

Since the motor is operating from a constant supply voltage, flux  $\phi$  is constant (neglecting armature reaction).

$$\therefore T_a \propto I_a$$

Hence  $T_a/I_a$  characteristic is a straight line passing through the origin as shown in Fig. The shaft torque ( $T_{sh}$ ) is less than  $T_a$  and is shown by a dotted line. It is clear from the curve that a very large current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

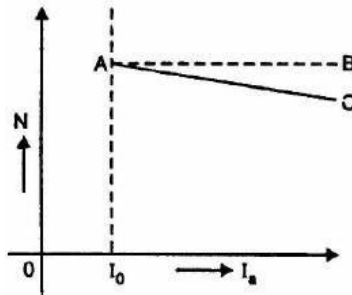


**(ii)  $N/I_a$  Characteristic.** The speed  $N$  of a d.c. motor is given by;

$$N \propto \frac{E_b}{\phi}$$

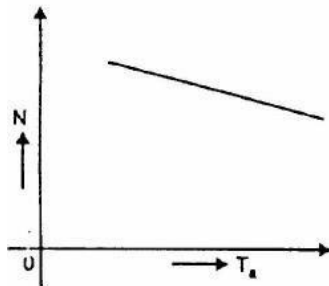
The flux  $\phi$  and back e.m.f.  $E_b$  in a shunt motor are almost constant under normal conditions. Therefore, speed of a shunt motor will remain constant as the armature current varies (dotted line AB

in Fig.). When load is increased,  $E_b (= V - I_a R_a)$  and  $\phi$  decrease due to the armature resistance drop and armature reaction respectively. However,  $E_b$  decreases slightly more than  $\phi$  so that the speed of the motor decreases slightly with load (line AC).



**(iii) N/T<sub>a</sub> Characteristic.**

The curve is obtained by plotting the values of N and T<sub>a</sub> for various armature currents. It may be seen that speed falls somewhat as the load torque increases.



**Conclusions**

Following two important conclusions are drawn from the above characteristics:

- (i) There is slight change in the speed of a shunt motor from no-load to fullload. Hence, it is essentially a constant-speed motor.
- (ii) The starting torque is not high because  $T_a \propto I_a$ .

**CHARACTERISTICS OF SERIES MOTORS**

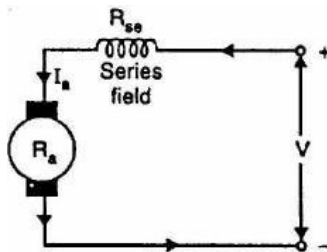


Fig. shows the connections of a series motor. Note that current passing through the field winding is the same as that in the armature. If the mechanical load on the motor increases, the armature current also increases. Hence, the flux in a series motor increases with the increase in armature current and vice-versa.

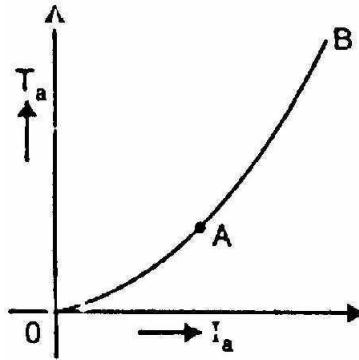
(i) **T<sub>a</sub>/I<sub>a</sub> Characteristic.** We know that:

$$T_a \propto \phi I_a$$

Up to magnetic saturation,  $\phi \propto I_a$  so that  $T_a \propto I_a^2$

After magnetic saturation,  $\phi$  is constant so that  $T_a \propto I_a$

Thus up to magnetic saturation, the armature torque is directly proportional to the square of armature current. If  $I_a$  is doubled,  $T_a$  is almost quadrupled.



Therefore,  $T_a/I_a$  curve upto magnetic saturation is a parabola (portion OA of the curve in Fig.). However, after magnetic saturation, torque is directly proportional to the armature current. Therefore,  $T_a/I_a$  curve after magnetic saturation is a straight line (portion AB of the curve).

It may be seen that in the initial portion of the curve (i.e. upto magnetic saturation),  $T_a \propto I_a^2$ . This means that starting torque of a d.c. series motor will be very high as compared to a shunt motor (where that  $T_a \propto I_a$ ).

**(ii)  $N/I_a$  Characteristic.**

The speed  $N$  of a series motor is given by;

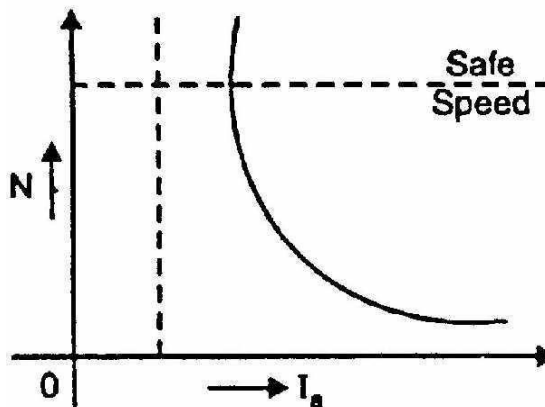
$$N \propto \frac{E_b}{\phi} \quad \text{where} \quad E_b = V - I_a(R_a + R_{se})$$

When the armature current increases, the back e.m.f.  $E_b$  decreases due to  $I_a(R_a + R_{se})$  drop while the flux  $\phi$  increases. However,  $I_a(R_a + R_{se})$  drop is quite small under normal conditions and may be neglected.

$$\therefore N \propto \frac{1}{\phi}$$

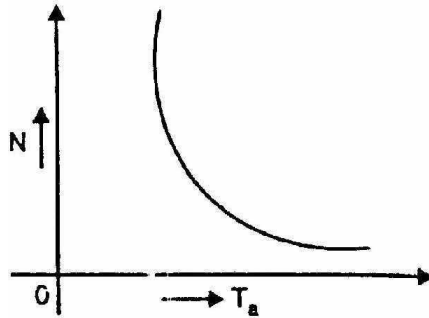
$$\propto \frac{1}{I_a} \text{ upto magnetic saturation}$$

Thus, up to magnetic saturation, the  $N/I_a$  curve follows the hyperbolic path as shown in Fig. After saturation, the flux becomes constant and so does the speed.



**(iii)  $N/T_a$  Characteristic.**

The  $N/T_a$  characteristic of a series motor is shown in Fig. It is clear that series motor develops high torque at low speed and vice-versa. It is because an increase in torque requires an increase in armature current, which is also the field current. The result is that flux is strengthened and hence the speed drops ( $\square N \propto 1/\phi$ ). Reverse happens should the torque be low.



**Conclusions**

- (i) It has a high starting torque because initially  $T_a \propto I_a^2$ .
- (ii) It is a variable speed motor ( $N/I_a$  curve) i.e., it automatically adjusts the speed as the load changes. Thus if the load decreases, its speed is automatically raised and vice-versa.
- (iii) At no-load, the armature current is very small and so is the flux. Hence, the speed rises to an excessive high value ( $\square N \propto 1/\phi$ ). This is dangerous for the machine which may be destroyed due to centrifugal forces set up in the rotating parts. Therefore, a series motor should never be started on no-load. However, to start a series motor, mechanical load is first put and then the motor is started.

**Note.** The minimum load on a d.c. series motor should be great enough to keep the speed within limits. If the speed becomes dangerously high, then motor must be disconnected from the supply.

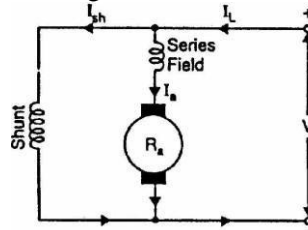
**Compound Motors**

A compound motor has both series field and shunt field. The shunt field is always stronger than the series field. Compound motors are of two types:

- (i) *Cumulative-compound motors* in which series field aids the shunt field.
  - (ii) *Differential-compound motors* in which series field opposes the shunt field.
- Differential compound motors are rarely used due to their poor torque characteristics at heavy loads.

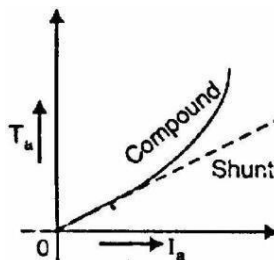
**Characteristics of Cumulative Compound Motors**

Fig. shows the connections of a cumulative-compound motor. Each pole carries a series as well as shunt field winding; the series field aiding the shunt field.



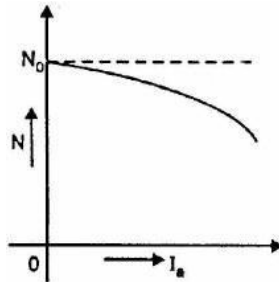
**(i)  $T_a/I_a$  Characteristic.**

As the load increases, the series field increases but shunt field strength remains constant. Consequently, total flux is increased and hence the armature torques ( $\square T_a \propto \phi I_a$ ). It may be noted that torque of a cumulative-compound motor is greater than that of shunt motor for a given armature current due to series field.



**(ii) N/I<sub>a</sub> Characteristic.**

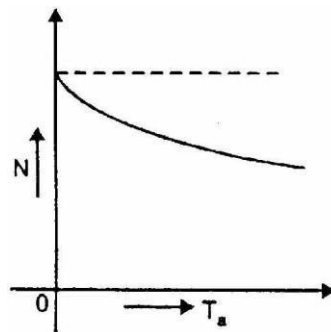
As explained above, as the load increases, the flux per pole also increases. Consequently, the speed ( $N \propto 1/\phi$ ) of the motor falls as the load increases (See Fig.). It may be noted that as the load is added, the increased amount of flux causes the speed to decrease more than does the speed of a shunt motor. Thus the speed regulation of a cumulative compound motor is poorer than that of a shunt motor.



**Note:** Due to shunt field, the motor has a definite no load speed and can be operated safely at no-load.

**(iii) N/T<sub>a</sub> Characteristic.**

Fig. shows N/T<sub>a</sub> characteristic of a cumulative compound motor. For a given armature current, the torque of a cumulative compound motor is more than that of a shunt motor but less than that of a series motor.

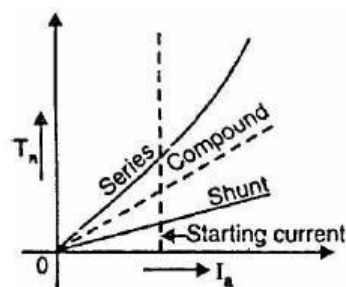
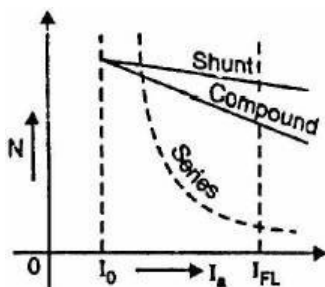


**Conclusions**

A cumulative compound motor has characteristics intermediate between series and shunt motors.

- (i) Due to the presence of shunt field, the motor is prevented from running away at no-load.
- (ii) Due to the presence of series field, the starting torque is increased.

**Comparison of Three Types of Motors**



- (i) The speed regulation of a shunt motor is better than that of a series motor. However, speed regulation of a cumulative compound motor lies between shunt and series motors.
- (ii) For a given armature current, the starting torque of a series motor is more than that of a shunt motor. However, the starting torque of a cumulative compound motor lies between series and shunt motors.
- (iii) Both shunt and cumulative compound motors have definite no-load speed. However, a series motor has dangerously high speed at no-load.

## APPLICATIONS OF D.C. MOTORS

### 1. Shunt motors

The characteristics of a shunt motor reveal that it is an approximately constant speed motor. It is, therefore, used

- (i) where the speed is required to remain almost constant from no-load to full-load
- (ii) where the load has to be driven at a number of speeds and any one of which is required to remain nearly constant

*Industrial use:* Lathes, drills, boring mills, shapers, spinning and weaving machines etc.

### 2. Series motors

It is a variable speed motor i.e., speed is low at high torque and vice-versa. However, at light or no-load, the motor tends to attain dangerously high speed. The motor has a high starting torque. It is, therefore, used

- (i) where large starting torque is required e.g., in elevators and electric traction
- (ii) where the load is subjected to heavy fluctuations and the speed is automatically required to reduce at high torques and vice-versa

*Industrial use:* Electric traction, cranes, elevators, air compressors, vacuum cleaners, hair drier, sewing machines etc.

### 3. Compound motors

Differential-compound motors are rarely used because of their poor torque characteristics. However, cumulative-compound motors are used where a fairly constant speed is required with irregular loads or suddenly applied heavy loads.

*Industrial use:* Presses, shears, reciprocating machines etc.

## Necessity of D.C. Motor Starter

At starting, when the motor is stationary, there is no back e.m.f. in the armature. Consequently, if the motor is directly switched on to the mains, the armature will draw a heavy current ( $I_a = V/R_a$ ) because of small armature resistance.

As an example, 5 H.P., 220 V shunt motor has a full-load current of 20 A and an armature resistance of about 0.5  $\Omega$ . If this motor is directly switched on to supply, it would take an armature current of  $220/0.5 = 440$  A which is 22 times the full-load current. This high starting current may result in:

- (i) burning of armature due to excessive heating effect,
- (ii) damaging the commutator and brushes due to heavy sparking,
- (iii) excessive voltage drop in the line to which the motor is connected. The result is that the operation of other appliances connected to the line may be impaired and in particular cases, they may refuse to work.

In order to avoid excessive current at starting, a variable resistance (known as starting resistance) is inserted in series with the armature circuit. This resistance is gradually reduced as the motor gains speed (and hence  $E_b$  increases) and eventually it is cut out completely when the motor has attained full speed. The value of starting resistance is generally such that starting current is limited to 1.25 to 2 times the full-load current.

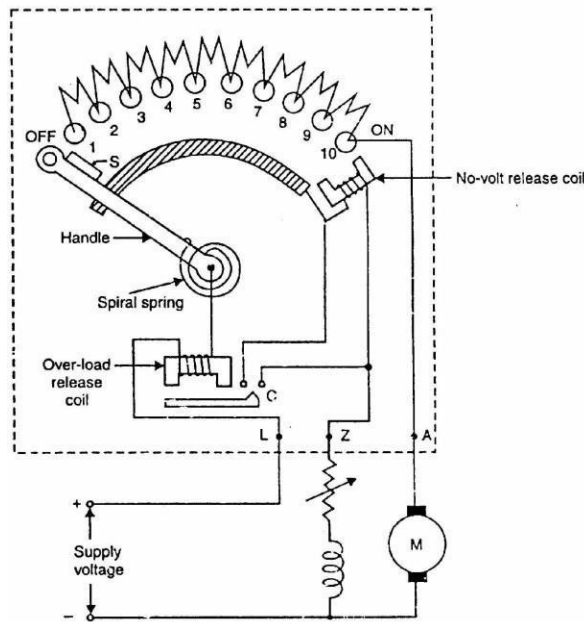
### Types of D.C. Motor Starters

The stalling operation of a d.c. motor consists in the insertion of external resistance into the armature circuit to limit the starting current taken by the motor and the removal of this resistance in steps as the motor accelerates. When the motor attains the normal speed, this resistance is totally cut out of the armature circuit. It is very important and desirable to provide the starter with protective devices to enable the starter arm to return to OFF position

- (i) when the supply fails, thus preventing the armature being directly across the mains when this voltage is restored. For this purpose, we use no-volt release coil.
- (ii) when the motor becomes overloaded or develops a fault causing the motor to take an excessive current. For this purpose, we use overload release coil.

### 1. Three-Point Starter

This type of starter is widely used for starting shunt and compound motors.



- It is so called because it has three terminals L, Z and A.
- The starter consists of starting resistance divided into several sections and connected in series with the armature.
- The tapping points of the starting resistance are brought out to a number of studs.
- The three terminals L, Z and A of the starter are connected respectively to the positive line terminal, shunt field terminal and armature terminal.
- The other terminals of the armature and shunt field windings are connected to the negative terminal of the supply.
- The no-volt release coil is connected in the shunt field circuit.
- One end of the handle is connected to the terminal L through the over-load release coil.
- The other end of the handle moves against a spiral spring and makes contact with each stud during starting operation, cutting out more and more starting resistance as it passes over each stud in clockwise direction.

#### Operation

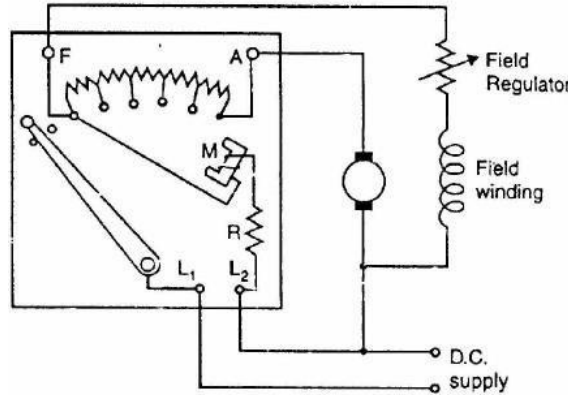
- To start with, the d.c. supply is switched on with handle in the OFF position.
- The handle is now moved clockwise to the first stud. As soon as it comes in contact with the first stud, the shunt field winding is directly connected across the supply, while the whole starting resistance is inserted in series with the armature.
- As the handle is gradually moved over to the final stud, the starting resistance is cut out of the armature circuit in steps. The handle is now held magnetically by the no-volt release coil which is energized by shunt field current.
- If the supply voltage is suddenly interrupted or if the field excitation is accidentally cut, the no-volt release coil is demagnetized and the handle goes back to the OFF position under the pull of the spring. If no-volt release coil were not used, then in case of failure of supply, the handle would remain on the final stud. If then supply is restored, the motor will be directly connected across the supply, resulting in an excessive armature current.
- If the motor is over-loaded (or a fault occurs), it will draw excessive current from the supply. This current will increase the ampere-turns of the over-load release coil and pull the armature C, thus short-circuiting the no-volt release coil. The no-volt coil is demagnetized and the handle is pulled to the OFF position by the spring. Thus, the motor is automatically disconnected from the supply.

**Drawback**

In a three-point starter, the no-volt release coil is connected in series with the shunt field circuit so that it carries the shunt field current. While exercising speed control through field regulator, the field current may be weakened to such an extent that the no-volt release coil may not be able to keep the starter arm in the ON position. This may disconnect the motor from the supply when it is not desired. This drawback is overcome in the four point starter.

**2. Four-Point Start**

In a four-point starter, the no-volt release coil is connected directly across the supply line through a protective resistance R. Fig. shows the schematic diagram of a 4-point starter for a shunt motor (over-load release coil omitted for clarity of the figure).



Now the no-volt release coil circuit is independent of the shunt field circuit. Therefore, proper speed control can be exercised without affecting the operation of no volt release coil.

Only difference between a three-point starter and a four-point starter is the manner in which no-volt release coil is connected. However, the working of the two starters is the same. It may be noted that the three point starter also provides protection against an open field circuit. This protection is not provided by the four-point starter.

**EFFICIENCY OF A D.C. MACHINE**

The power that a d.c. machine receives is called the input and the power it gives out is called the output. Therefore, the efficiency of a d.c. machine, like that of any energy-transferring device, is given by;

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \tag{i}$$

$$\text{Output} = \text{Input} - \text{Losses} \quad \text{and} \quad \text{Input} = \text{Output} + \text{Losses}$$

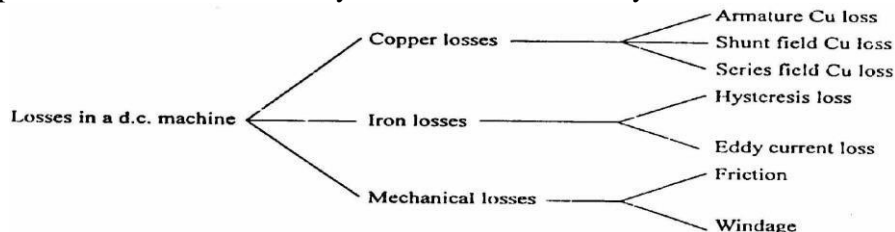
Therefore, the efficiency of a d.c. machine can also be expressed in the following forms:

$$\text{Efficiency} = \frac{\text{Input} - \text{Losses}}{\text{Input}} \tag{ii}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}} \tag{iii}$$

**Losses in a D.C. Machine**

The losses in a d.c. machine (generator or motor) may be divided into three classes viz (i) **copper losses** (ii) **iron or core losses** and (iii) **mechanical losses**. All these losses appear as heat and thus raise the temperature of the machine. They also lower the efficiency of the machine.





### 1. Copper losses

These losses occur due to currents in the various windings of the machine.

- (i) Armature copper loss =  $I_a^2 R_a$
- (ii) Shunt field copper loss =  $I_{sh}^2 R_{sh}$
- (iii) Series field copper loss =  $I_{se}^2 R_{se}$

### 2. Iron or Core losses

These losses occur in the armature of a d.c. machine and are due to the rotation of armature in the magnetic field of the poles. They are of two types viz., (i) hysteresis loss (ii) eddy current loss.

### 3. Mechanical losses

These losses are due to friction and windage.

- (i) friction loss e.g., bearing friction, brush friction etc.
- (ii) windage loss i.e., air friction of rotating armature.

These losses depend upon the speed of the machine. But for a given speed, they are practically constant.

**Note.** Iron losses and mechanical losses together are called stray losses.

### Constant and Variable Losses

The losses in a d.c. generator (or d.c. motor) may be sub-divided into (i) constant losses (ii) variable losses.

#### (i) Constant losses

Those losses in a d.c. generator which remain constant at all loads are known as constant losses. The constant losses in a d.c. generator are:

- (a) iron losses
- (b) mechanical losses
- (c) shunt field losses

#### (ii) Variable losses

Those losses in a d.c. generator which vary with load are called variable losses.

The variable losses in a d.c. generator are:

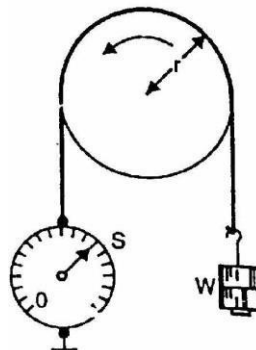
- (a) Copper loss in armature winding
- (b) Copper loss in series field winding

### Total losses = Constant losses + Variable losses

**Note.** Field Cu loss is constant for shunt and compound generators.

### Efficiency by Direct Loading (Load test)

In this method, the d.c. machine is loaded and output and input are measured to find the efficiency.



In this method, a brake is applied to a water-cooled pulley mounted on the motor shaft as shown in Fig. One end of the rope is fixed to the floor via a spring balance S and a known mass is suspended at the other end. If the spring balance reading is S kg-Wt and the suspended mass has a weight of W kg-Wt, then,

Net pull on the rope =  $(W - S)$  kg-Wt =  $(W - S) \times 9.81$  newtons

If  $r$  is the radius of the pulley in metres, then the shaft torque  $T_{sh}$  developed by the motor is

$$T_{sh} = (W - S) \times 9.81 \times r \text{ N - m}$$

If the speed of the pulley is  $N$  r.p.m., then,

$$\text{Output power} = \frac{2\pi N T_{sh}}{60} = \frac{2\pi N \times (W - S) \times 9.81 \times r}{60} \text{ watts}$$

Let  $V$  = Supply voltage in volts

$I$  = Current taken by the motor in amperes

$$\therefore \text{Input to motor} = V I \text{ watts}$$

$$\therefore \text{Efficiency} = \frac{2\pi N(W - S) \times r \times 9.81}{60 \times VI}$$

# 3

## Transformers

---

Electrical power transformer is a static device which transforms electrical energy from one circuit to another without change in frequency. Since there is no rotating or moving part in a transformer, it is a static device. Transformer can increase or decrease the voltage with corresponding decrease or increase in current. Basically a transformer consists of two inductive windings and a laminated steel core. The coils are insulated from each other as well as from the soft iron or silicon steel core.

Winding connected to the AC supply is called primary windings and the winding on which load is connected is called secondary windings.

Transformer is not an electromechanical energy conversion device but an electro-magnetic device. When the primary windings is connected to the AC supply, the electrical energy is first converted to magnetic energy and then it is reconverted to electrical energy in the secondary windings.

If the secondary voltage is higher than primary voltage, then the secondary windings has more number of turns than in the primary and the transformer is called *step-up transformer*. Also if the secondary voltage is lower than primary voltage, then the secondary windings has less number of turns than in the primary and the transformer is called *step-down transformer*.



Figure 3.1: Transformer

The main function of a transformer is to Step up ( Increase) or Step down (Decrease) the level of voltage at the same time it decreases or increases the level of current, without any change in power and frequency.

Transmission of electric power in DC is very difficult and expensive with higher voltage levels using DC converters. For this purpose, we can use a transformer. Transformer is a highly efficient device due to which the electric power is transmitted and distributed in AC instead of DC. Transformer cannot work on DC.

### 3.1 Principle of Operation

The basic principle behind working of a transformer is the phenomenon of *mutual induction* between two windings linked by common magnetic flux. The Figure 3.2 shows the simplest form of a two winding transformer. Basically a transformer consists of two inductive coils which are electrically separated but magnetically linked through a path of low reluctance.

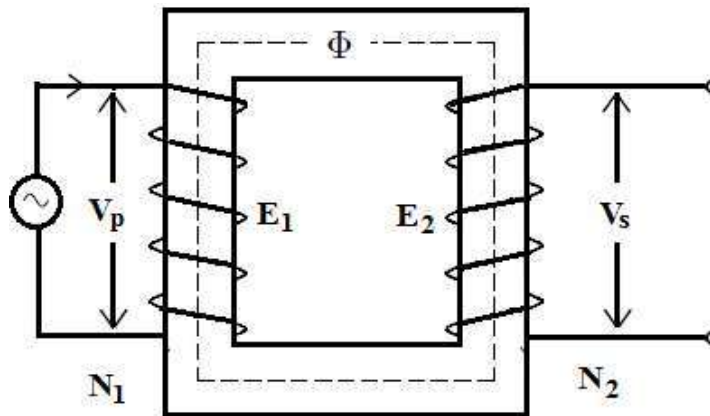


Figure 3.2: Two Winding Transformer

When the primary winding is connected to a source of alternating voltage, an alternating current starts flowing through the winding which produces an alternating flux  $\Phi$  in the core. The core provides magnetic path for the flux, to get linked with the secondary winding. As the flux produced is alternating (the direction of it is continuously changing), e.m.f. gets induced in the secondary winding according to Faraday's law of electromagnetic induction. This emf is called *mutually induced emf*, and the frequency of mutually induced emf is same as that of applied voltage. If the secondary winding is closed circuit, then mutually induced current flows through it, and hence the electrical energy is transferred from one circuit (primary) to another circuit (secondary).

Most of the flux gets linked with the secondary winding which is called as *useful flux* or *main flux*, and the flux which does not get linked with secondary winding is called as *leakage flux*. The following facts are to be noted:

## Transformers

- ☞ The transformer action is governed by the laws of electromagnetic induction.
- ☞ The primary and secondary windings are electrically isolated.
- ☞ The power is transferred from primary to secondary through magnetic flux.
- ☞ There is no change in frequency when the power is transferred from primary to secondary
- ☞ The transformer changes the voltage level

**Table 3.1: Comparison of Core type and Shell type Transformer**

	Core Type	Shell Type
1	The winding surround the considerable portion of core	The core surround the considerable portion of winding
2	winding is placed on two core limbs.	winding is placed on mid arm
3	Concentric winding or cylindrical Winding is used	Sandwich or Disc winding is used
4	One path of the magnetic circuit	Two path of the magnetic circuit
5	Used for large sized, low voltage transformer	Used for small size high voltage transformer
6	It has two limbs	It has three limbs
7	Less mechanical protection to coil	Better mechanical protection to coil
8	Easy to repair	Not easy to repair
9	Transformer losses are more	Transformer losses are more less
10	Better natural cooling	Natural Cooling is not that effective

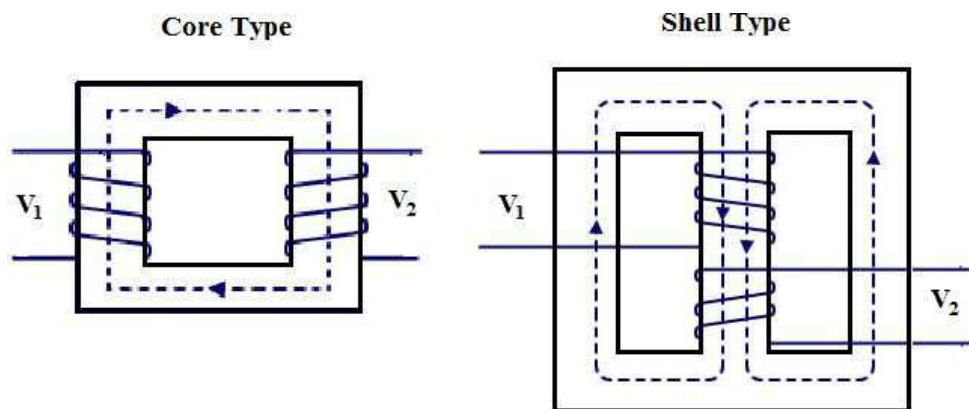


Figure 3.3: Core Type and Shell Type Transformer

## Transformers

### 3.2 Ideal Transformer

An ideal transformer consists of two purely inductive coils wound on a loss-free core. It is not possible to design such a transformer in practice but we will discuss about that before we start discussing about practical transformer. An ideal transformer has the following characteristics,

- (i) No winding resistance
- (ii) No leakage flux i.e., the same flux links with both windings
- (iii) Core losses (Eddy current and hysteresis losses) are absent in the core.
- (iv) Infinite permeability of the core

Consider an ideal transformer whose secondary is open-circuited as shown in Figure 3.4(a). When an alternating voltage  $V_1$  is applied to the primary, it draws a small magnetizing current  $I_m$  which lags behind the applied voltage by  $90^\circ$ . This alternating current  $I_m$  produces an alternating flux  $\Phi$  which is proportional and also in phase with it. The alternating flux  $\Phi$  links with both the windings. It produces a self induced e.m.f.  $E_1$  in the primary winding and mutually induced e.m.f.  $E_2$  in the secondary winding. The primary e.m.f.  $E_1$  and secondary e.m.f.  $E_2$  are inphase and they are antiphase with  $V_1$ . However, their magnitudes depend upon the number of primary and secondary turns.

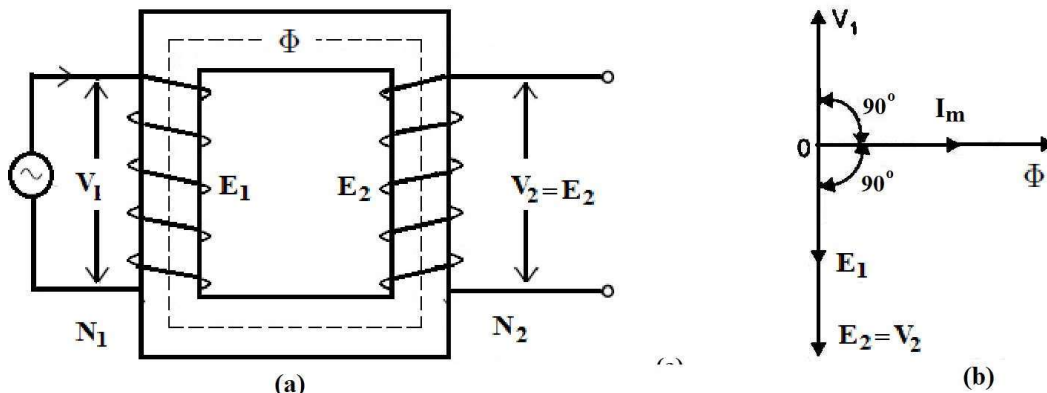


Figure 3.4: Ideal transformer

Figure 3.4(b) shows the phasor diagram of an ideal transformer on no load. Since flux  $\Phi$  is common to both primary and secondary windings, it has been taken as the reference phasor. The current  $I_m$  is the magnetising current which is inphase with the flux  $\Phi$ . The primary e.m.f.  $E_1$  and secondary e.m.f.  $E_2$  are inphase and lag behind the flux  $\Phi$  by  $90^\circ$ . The primary e.m.f.  $E_1$  and secondary e.m.f.  $E_2$  are antiphase with the applied voltage  $V_1$ .

### 3.3 Emf Equation

When the primary winding of a transformer is applied with a sinusoidal voltage, an alternating flux set up in the core which links with both primary and secondary windings. Figure 3.5 shows that the flux rises sinusoidally and it reaches to the maximum value in one quarter of the cycle i.e in  $T/4$  sec (where,  $T$  is time period of the sine wave of the supply whose frequency is  $1/f$ ).

Let,

- $\Phi_m$  = Maximum flux in the core in weber
- $f$  = Frequency of input in Hertz
- $N_1$  = Number of turns in the primary
- $N_2$  = Number of turns in the secondary
- $B_m$  = Maximum flux density in Tesla
- $A$  = Effective area of core in  $m^2$

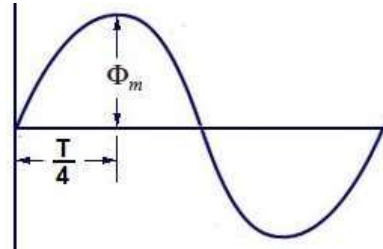


Figure 3.5: Variation of flux

$$\begin{aligned} \text{Average rate of change of flux} &= \frac{d\Phi}{dt} \\ &= \frac{\Phi_m}{1/4f} \\ &= 4f\Phi_m \text{ Volt} \end{aligned}$$

Average rate of change of flux is equal to Average emf induced per turn

$$\therefore \text{Average emf induced per turn} = 4f\Phi_m \text{ Volt}$$

Since the alternating flux varies sinusoidally and the form factor of sinusoidal wave is 1.11, then the r.m.s. value of induced emf is 1.11 times average value

$$\therefore \text{r.m.s. value of induced emf per turn} = 4.44f\Phi_m \text{ Volt}$$

r.m.s. value of induced emf in the primary winding

$$\begin{aligned} E_1 &= 4.44 \Phi_m f N_1 \text{ Volt} \\ &= 4.44 B_m A f N_1 \text{ Volt} \end{aligned} \tag{3.1}$$

r.m.s. value of induced emf in the secondary winding

$$\begin{aligned} E_2 &= 4.44 \Phi_m f N_2 \text{ Volt} \\ &= 4.44 B_m A f N_2 \text{ Volt} \end{aligned} \tag{3.2}$$

From equation 3.1 and equation 3.2

$$\frac{E_2}{N_2} = \frac{E_1}{N_1} = 4.44 B_m A f \tag{3.3}$$

$\therefore$  For a transformer emf/turn is same for both primary and secondary windings.



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### 3.4 Voltage transformation ratio

From Section 3.3, emf/turn is same for both primary and secondary windings. So in a transformer, the ratio of secondary and primary induced voltage and the ratio of secondary and primary number of turns is equal to a constant. That constant is known as Voltage transformation ratio  $K$ .

From equation 3.1 and equation 3.2

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K \quad (3.4)$$

Where the constant  $K$  is known as the voltage transformation ratio.

For an ideal transformer, if  $V_1$  is the applied voltage and  $V_2$  is the terminal voltage,

$$V_1 = E_1 \text{ and } V_2 = E_2$$

Also for an ideal transformer,

$$\text{Input VA} = \text{Output VA}$$

$$V_1 I_1 = V_2 I_2 \quad (3.5)$$

$$\therefore \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K \quad (3.6)$$

☞ For a step-up transformer  $N_2 > N_1$ , then  $K > 1$

☞ For a step-down transformer  $N_2 < N_1$ , then  $K < 1$

#### Example 3. 1

A 50 KVA single phase transformer has 500 turns in the primary and 50 turns in the secondary. The primary side is connected to 11 KV, 50 Hz supply. Determine (i) no load secondary voltage (ii) primary and secondary currents at full load.

**Solution:** (i)

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

(i) No load secondary voltage

$$\begin{aligned} V_2 &= \frac{N_2 V_1}{N_1} \\ &= \frac{50 \times 11000}{500} \\ &= 1100 \text{ V} \\ &= 1.1 \text{ KV} \end{aligned}$$

(ii) Now, Primary current,

$$\begin{aligned}
 I_1 &= \frac{KVA \times 1000}{V_1} \\
 &= \frac{50 \times 1000}{11000} \\
 &= 4.55 \text{ A}
 \end{aligned}$$

Secondary current,

$$\begin{aligned}
 I_2 &= \frac{KVA \times 1000}{V_2} \\
 &= \frac{50 \times 1000}{11000} \\
 &= 45.45 \text{ A}
 \end{aligned}$$

**Example 3. 2**

A 100 kVA, 11000/400 V, 50-Hz, single phase, core type transformer has a core of cross-section of 10 cm × 10 cm. Find (i) the number of H.V. and L.V. turns per phase and (ii) the e.m.f. per turn if the maximum core density is not to exceed 1.4 T.

**Solution:**

Maximum flux density,  $B_m = 1.4 \text{ T}$

Area of cross section,  $A = 0.1 \times 0.1$   
 $= 0.01 \text{ m}^2$

(i) We know that,

$$E_1 = 4.44 B_m A f N_1$$

$$\begin{aligned}
 \therefore N_1 &= \frac{E_1}{4.44 B_m A f} \\
 &= \frac{11000}{4.44 \times 1.4 \times 0.01 \times 50} \\
 &= 3540
 \end{aligned}$$

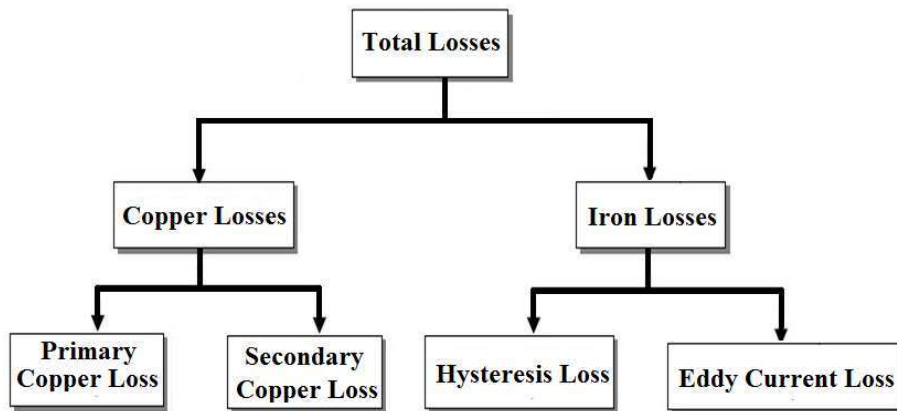
$$\begin{aligned}
 N_2 &= \frac{E_2}{4.44 B_m A f} \\
 &= \frac{400}{4.44 \times 1.4 \times 0.01 \times 50} \\
 &= 129
 \end{aligned}$$

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(ii)

$$\begin{aligned} \text{emf/turn} &= \frac{11000}{3540} \\ &= 3.11V \end{aligned}$$

### 3.5 Losses and Efficiency



#### 3.5.1 Copper losses

The copper losses or electrical losses are the winding losses taking place due to the flow of current through primary and secondary windings. These losses depend upon the resistance in the windings. The total copper loss is equal to the sum of copper losses occurring in the primary and the secondary windings. Copper losses can be experimentally determined by conducting short circuit test.

If  $I_1$  and  $I_2$  are the primary and secondary currents respectively and  $R_1$  and  $R_2$  are the primary and secondary resistances respectively, then

$$\text{Total Copper Loss} = I_1^2 R_1 + I_2^2 R_2$$

#### 3.5.2 Iron losses

Iron losses are also called as core losses or magnetic losses. This losses occur in the transformer core which is made up of iron. It is caused by the alternating flux in the core. Iron losses are considered as constant losses. Iron losses occurring in a transformer can be classified as hysteresis loss and eddy current loss.

- (a) **Hysteresis loss** : Hysteresis losses occur in the core due to the alternating flux. It is due to the reversal of magnetization of the core. The core of the transformer is subjected to an alternating magnetic flux. Each time the magnetic field is reversed, a small amount of energy is lost due to hysteresis within the core. For a given core material, the transformer losses are proportional to the frequency,

and is a function of the peak flux density to which it is subjected. The hysteresis loss is directly proportional to the number of magnetic reversal per second. The Steinmetz formula is used for the calculation of hysteresis loss and is shown below.

$$\text{Hysteresis loss, } P_h = \eta B_{max}^{1.6} f v \quad \text{Watts}$$

where  $\eta$  is the Steinmetz hysteresis co-efficient,  $B_{max}$  is the maximum flux density,  $f$  is the frequency of reversal and  $v$  is the Volume of the core.

- (b) **Eddy current loss** : When the iron core is subjected to varying magnetic field, an emf is also induced in the core according to Faraday's law of electromagnetic induction. Though this induced emf is small which give rise to the circulation of large current known as eddy current due to the low resistance of the core. The power loss due to this current is known as eddy current loss. This loss is almost constant for the transformer. In order to reduce eddy current loss, the transformer core is made up of thin laminations insulated from each other. Eddy current loss can be determined using the following equation.

$$\text{Eddy current loss, } P_e = K_e B_{max}^2 f^2 v t^2 \quad \text{Watts}$$

where  $K_e$  is a constant,  $B_{max}$  is the maximum flux density,  $f$  is the frequency of reversal,  $v$  is the volume of the transformer core and  $t$  is the thickness of the lamination.

### 3.6 Efficiency of Transformer

The ratio of output power to input power is called the efficiency of the transformer. It is expressed as

$$\begin{aligned} \text{Efficiency, } \eta &= \frac{\text{Output power}}{\text{Input power}} \\ &= \frac{\text{Output power}}{\text{Output power} + \text{Losses}} \\ &= \frac{\text{Input power} - \text{Losses}}{\text{Input power}} \\ &= 1 - \frac{\text{Losses}}{\text{Input power}} \end{aligned}$$

## Transformers

### 3.7 Condition for Maximum Efficiency

Consider the primary side of the transformer,

$$\text{Copper Loss} = I_1^2 R_{01} = I_2^2 R_{02}$$

$$\text{Transformer Input} = V_1 I_1 \cos \phi_1$$

$$\text{Transformer Output} = \text{Input} - \text{Losses}$$

$$= \text{Input} - \text{Copper loss} - \text{Iron losses}$$

$$= V_1 I_1 \cos \phi_1 - I_1^2 R_{01} - W_i$$

$$\begin{aligned} \therefore \text{Efficiency, } \eta &= \frac{\text{Output power}}{\text{Input power}} \\ &= \frac{V_1 I_1 \cos \phi_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos \phi_1} \\ &= 1 - \frac{I_1 R_{01}}{V_1 \cos \phi_1} - \frac{W_i}{V_1 I_1 \cos \phi_1} \end{aligned}$$

Differentiating both sides with respect to  $I_1$

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos \phi_1} - \frac{W_i}{V_1 I_1^2 \cos \phi_1}$$

For maximum efficiency  $\frac{d\eta}{dI_1} = 0$

$$\begin{aligned} \frac{R_{01}}{V_1 \cos \phi_1} &= \frac{W_i}{V_1 I_1^2 \cos \phi_1} \\ \therefore I_1^2 R_{01} &= W_i \end{aligned}$$

Hence the condition for maximum efficiency is,

$$\boxed{\text{Copper loss} = \text{Core loss}}$$

#### Example 3. 3

In a 20-kVA, 2200/400 V, single-phase transformer, the iron and full-load copper losses are 300 and 400 W respectively. Calculate the efficiency at unity power factor on (i) full load (ii) half full-load.

**Solution :**

(i) Full-load Unity p.f.

$$\begin{aligned} \text{Total loss} &= 300 + 400 \\ &= 700 \text{ W} = 0.7 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{F.L. output at u.p.f} &= 20 \times 1 \\ &= 20 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Input} &= 20 + 0.7 \\ &= 20.7 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Efficiency, } \eta &= \frac{20}{20.7} \times 100 \\ &= 97.3\% \end{aligned}$$

(ii) Half F.L. Unity p.f.

$$\begin{aligned} \text{Cu loss} &= 400 \times (1/2)^2 \\ &= 100 \text{ W} \end{aligned}$$

Iron loss remains constant at 300 W,

$$\begin{aligned} \text{Total loss} &= 100 + 300 \\ &= 400 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Half-load output at u.p.f} &= 20 \times \frac{1}{2} \text{ kW} \\ &= 10 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Input} &= 10 + 0.4 \\ &= 10.4 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Efficiency, } \eta &= \frac{10}{10.4} \times 100 \\ &= 96.15\% \end{aligned}$$

**Example 3. 4**

A 10-kVA, 440/220 V, 1- $\phi$ , 50 Hz transformer has iron loss of 350 W. The Cu loss is found to be 100 W when delivering half full-load current. Determine the efficiency when delivering full-load current at 0.8 lagging p.f.

**Solution.**

$$\text{Half load Cu loss} = 100 \text{ W}$$

$$\begin{aligned} \text{Full load Cu loss} &= 100 \times (2)^2 \\ &= 400 \text{ W} \end{aligned}$$

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Iron loss remains constant at 350 W,

$$\begin{aligned}\text{Total loss} &= 400 + 350 \\ &= 750 \text{ W} \\ &= 0.75 \text{ kW}\end{aligned}$$

$$\begin{aligned}\text{Full load output at } 0.8 \text{ lag} &= 10 \times 0.8 \text{ kW} \\ &= 8 \text{ kW}\end{aligned}$$

$$\begin{aligned}\text{Input} &= 8 + 0.75 \\ &= 8.75 \text{ kW}\end{aligned}$$

$$\begin{aligned}\text{Efficiency, } \eta &= \frac{8}{8.75} \times 100 \text{ ---} \\ &= 91.43\%\end{aligned}$$

### 3.8 Practical transformer

In practice no transformer is ideal. In this section we shall discuss about a practical transformer. The practical transformer has

- (i) Winding resistance
- (ii) Leakage flux which causes leakage reactance
- (iii) Core losses (eddy current and hysteresis losses) in the core.
- (iv) Finite permeability of the core.

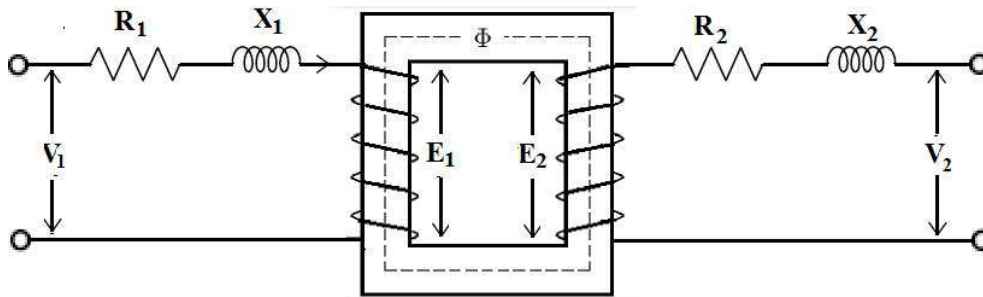


Figure 3.6: Practical Transformer

- (i) **Winding resistance** : Primary and secondary Windings consist of copper conductors. So primary and secondary side will have winding resistances. The primary resistance  $R_1$  and secondary resistance  $R_2$  are connected in series with the respective windings as shown in Figure 3.6. When current flows through the windings, there will be power loss  $I^2R$  as well as a loss in voltage due to  $IR$  drop.

- (ii) **Leakage Reactance :** When current flows through the primary and secondary windings, flux is produced. The part of flux  $\Phi$  which links both the windings is the useful flux and is called mutual flux and the part of flux  $\Phi$  which would not link the secondary winding is called leakage flux. The primary leakage flux can be represented as inductive reactance  $X_1$  and the secondary leakage flux can be represented as inductive reactance  $X_2$ . However, the presence of leakage reactance in the primary and secondary windings may change the power factor and causes voltage drop  $IX$ .
  
- (iii) **Core losses :** When the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. We have already discussed about this in the section 3.5
  
- (iv) **Finite permeability of the core :** The reluctance of the magnetic circuit depends not only on the length and cross sectional area, but also on the permeability ( $\mu$ ) of the material. The higher the value for  $\mu$  the more flux will flow to the secondary. Ideal transformer has infinite permeability that means no leakage flux. In practical transformer the flux linkage depends on the value of permeability of the core

### 3.9 Transformer on No load

When a practical transformer on no load i.e., secondary on open-circuit as shown in Figure 3.7. there is iron loss in the core and copper loss in both primary and secondary windings. These losses are very small. The primary will draw a small current  $I_0$  to supply (i) the iron losses and (ii) a very small amount of copper loss in the primary. Hence the primary no load current  $I_0$  is not  $90^\circ$  behind the applied voltage  $V_1$  but lags it by an angle  $\phi_0 < 90^\circ$  as shown in the phasor diagram in Figure 3.7(b).

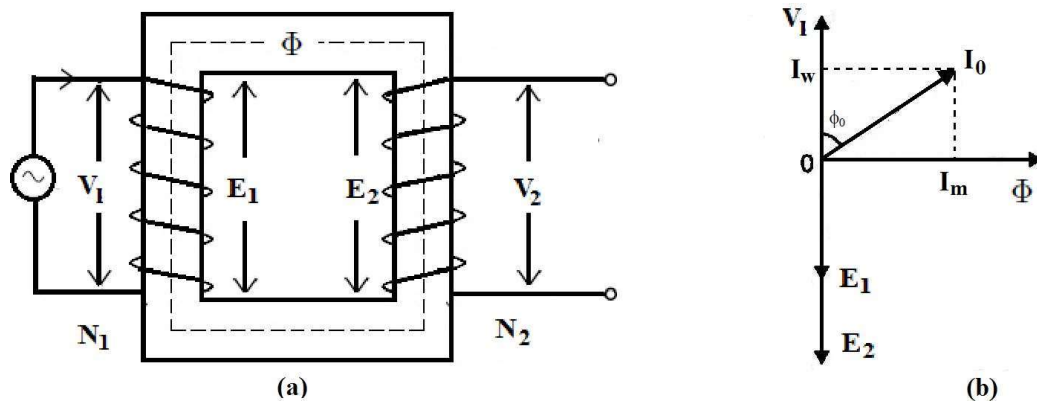


Figure 3.7: Transformer on No load



## Transformers

Noload input power,

$$W_0 = V_1 I_0 \cos \varphi_0 \quad (3.7)$$

The core-loss which is responsible for shift in the no-load current through an angle  $\varphi_0$  so it is known as hysteresis angle of advance.

$\therefore$  No-load power factor,

$$\cos \varphi_0 = \frac{W_0}{V_1 I_0} \quad (3.8)$$

The no-load primary current  $I_0$  can be resolved into two rectangular components. The component  $I_w$  in phase with the applied voltage  $V_1$ . This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$I_w = I_0 \cos \varphi_0 \quad (3.9)$$

The component  $I_m$  lagging behind  $V_1$  by  $90^\circ$  and is known as magnetizing component. It is this component which produces the mutual flux  $\Phi$  in the core.

$$I_m = I_0 \sin \varphi_0 \quad (3.10)$$

$$\therefore I_0 = \sqrt{I_m^2 + I_w^2} \quad (3.11)$$

The no load primary copper loss is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer.

No load input power,  $W_0 = \text{Iron loss}$

### 3.10 Vector Diagrams of transformer on Load

Practical transformer has winding resistance and leakage reactance. There is voltage drop in  $R_1$  and  $X_1$  so that primary induced e.m.f.  $E_1$  is less than the applied voltage  $V_1$ . Similarly, there is voltage drop in  $R_2$  and  $X_2$  so that secondary terminal voltage  $V_2$  is less than the secondary induced e.m.f.  $E_2$ .

When the transformer is loaded, the secondary current  $I_2$  will flow. The magnitude and phase of  $I_2$  with respect to  $V_2$  is depending upon the type of load connected on the secondary side of the transformer. An additional primary current be  $I_2'$  will flow in the primary side when the transformer is loaded. It is known as load component of primary current. The additional primary m.m.f.  $N_1 I_2'$  sets up its own flux  $\varphi_2'$  which is equal and opposition to  $\varphi_2$ . So the two fluxes cancel each other. Hence, the net flux passing through the core is approximately the same as at no-load and due to this, the core loss is also practically constant for all conditions.

In this section we shall discuss about the vector diagrams of a transformer, when connected to resistive load, inductive load and capacitive load.

### 3.10.1 Resistive Load

When the transformer is connected to a resistive load, the output voltage  $V_2$  and the secondary current  $I_2$  are inphase. Figure 3.8 shows the phasor diagram of a practical transformer for the resistive load. the primary and secondary induced e.m.f  $E_1$  and  $E_2$  lags behind the mutual flux  $\Phi$  by an angle  $90^\circ$ . As the secondary side is loaded, an additional current  $I_2'$  flows in the primary side which is anti-phase with the secondary current  $I_2$  and K times its magnitude. Also  $I_0$  is the no-load current of the transformer. The vector sum of  $I_2'$  and  $I_0$  gives the total primary current  $I_1$ , which is at an angle  $\phi_1$  from  $V_1$ .

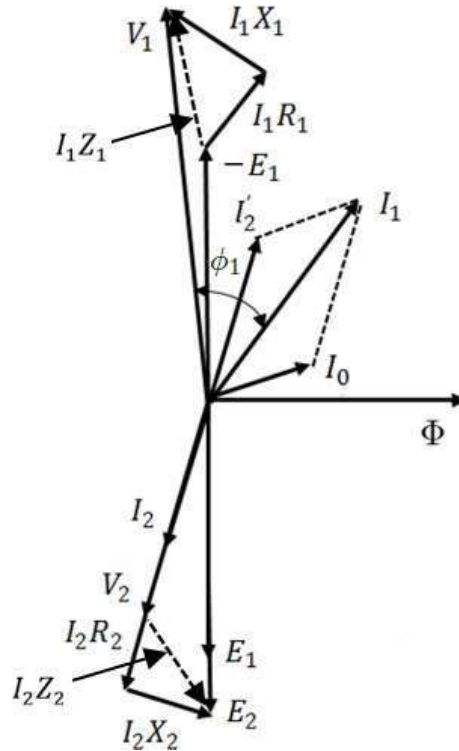


Figure 3.8: .

Now the induced e.m.f. in the primary side that opposes the applied voltage  $V_1$  is  $E_1$  and is drawn opposite to  $E_1$  as  $-E_1$ . The resistance voltage drop across the primary winding  $I_1R_1$  is drawn in parallel to the primary current  $I_1$  and the reactance voltage drop in the primary winding  $I_1X_1$  is drawn in perpendicular to the primary current  $I_1$  from  $-E_1$ . The vector sum of  $I_1R_1$  and  $I_1X_1$  gives  $I_1Z_1$ . Also the vector sum of  $-E_1$  and  $I_1Z_1$  gives the applied primary voltage  $V_1$ .

In the secondary side, the phasor  $E_2$  represents the induced e.m.f. in the secondary by the mutual flux  $\Phi$ . The secondary terminal voltage is  $V_2$ . Now The resistance voltage drop across the secondary winding  $I_2R_2$  is drawn in parallel to the secondary current  $I_2$  and the reactance voltage drop in the secondary winding  $I_2X_2$

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is drawn in perpendicular to the secondary current  $I_2$  from  $V_2$ . The vector sum of  $I_2R_2$  and  $I_2X_2$  gives  $I_2Z_2$ . Also the vector sum of  $V_2$  and  $I_2Z_2$  gives the mutually induced e.m.f.  $E_2$  in the secondary side.

From the vector diagram, the primary impedance is given by,

$$Z_1 = R_1 + jX_1 \quad (3.12)$$

Similarly the secondary impedance is given by,

$$Z_2 = R_2 + jX_2 \quad (3.13)$$

In the primary side the induced e.m.f. is decreased because of the voltage drop due to resistive and reactance drop. the input voltage is given by,

$$\begin{aligned} V_1 &= -E_1 + I_1(R_1 + jX_1) \\ &= -E_1 + I_1Z_1 \end{aligned} \quad (3.14)$$

Similarly in the secondary side the terminal voltage is decreased because of the voltage drop due to resistive and reactance drop. the mutually induced e.m.f. is given by,

$$\begin{aligned} E_2 &= V_2 + I_2(R_2 + jX_2) \\ &= V_2 + I_2Z_2 \end{aligned} \quad (3.15)$$

### 3.10.2 Inductive Load

When the transformer is connected to an inductive load, the secondary current  $I_2$  lags behind the output voltage  $V_2$  by an angle  $\varphi_2$ . Figure 3.9 shows the phasor diagram of a practical transformer for the inductive load. the primary and secondary induced e.m.f  $E_1$  and  $E_2$  lags behind the mutual flux  $\Phi$  by an angle  $90^\circ$ . As the secondary side is loaded, an additional current  $I_2'$  flows in the primary side which is anti-phase with the secondary current  $I_2$  and  $K$  times its magnitude. Also  $I_0$  is the no-load current of the transformer. The vector sum of  $I_2'$  and  $I_0$  gives the total primary current  $I_1$  which is at an angle  $\varphi_1$  from  $V_1$ .

Now the induced e.m.f. in the primary side that opposes the applied voltage  $V_1$  is  $E_1$  and is drawn opposite to  $E_1$  as  $-E_1$ . The resistance voltage drop across the primary winding  $I_1R_1$  is drawn in parallel to the primary current  $I_1$  and the reactance voltage drop in the primary winding  $I_1X_1$  is drawn in perpendicular to the primary current  $I_1$  from  $-E_1$ . The vector sum of  $I_1R_1$  and  $I_1X_1$  gives  $I_1Z_1$ . Also the vector sum of  $-E_1$  and  $I_1Z_1$  gives the applied primary voltage  $V_1$ .

In the secondary side, the phasor  $E_2$  represents the induced e.m.f. in the secondary by the mutual flux  $\Phi$ . The secondary terminal voltage is  $V_2$ . Now The resistance voltage drop across the secondary winding  $I_2R_2$  is drawn in parallel to the secondary current  $I_2$  and the reactance voltage drop in the secondary winding  $I_2X_2$  is drawn in perpendicular to the secondary current  $I_2$  from  $V_2$ . The vector sum of

$I_2R_2$  and  $I_2X_2$  gives  $I_2Z_2$ . Also the vector sum of  $V_2$  and  $I_2Z_2$  gives the mutually induced e.m.f.  $E_2$  in the secondary side.

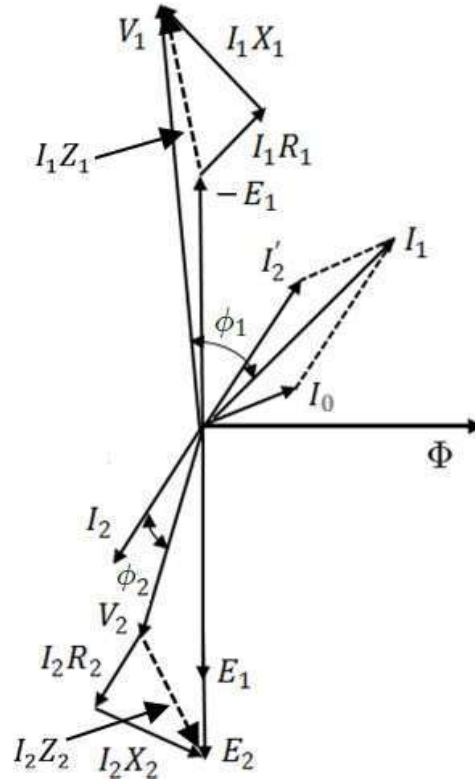


Figure 3.9: •

From the vector diagram, the primary impedance is given by,

$$Z_1 = R_1 + jX_1 \quad (3.16)$$

Similarly the secondary impedance is given by,

$$Z_2 = R_2 + jX_2 \quad (3.17)$$

In the primary side the induced e.m.f. is decreased because of the voltage drop due to resistive and reactance drop. the input voltage is given by,

$$\begin{aligned} V_1 &= -E_1 + I_1(R_1 + jX_1) \\ &= -E_1 + I_1Z_1 \end{aligned} \quad (3.18)$$

Similarly in the secondary side the terminal voltage is decreased because of the voltage drop due to resistive and reactance drop. the mutually induced e.m.f. is given by,

$$\begin{aligned} E_2 &= V_2 + I_2(R_2 + jX_2) \\ &= V_2 + I_2Z_2 \end{aligned} \quad (3.19)$$

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### 3.10.3 Capacitive Load

When the transformer is connected to a capacitive load, the secondary current  $I_2$  leads the output voltage  $V_2$  by an angle  $\phi_2$ . Figure 3.10 shows the phasor diagram of a practical transformer for the capacitive load. The primary and secondary induced e.m.f.  $E_1$  and  $E_2$  lags behind the mutual flux  $\Phi$  by an angle  $90^\circ$ . As the secondary side is loaded, an additional current  $I_2'$  flows in the primary side which is anti-phase with the secondary current  $I_2$  and  $K$  times its magnitude. Also  $I_0$  is the no-load current of the transformer. The vector sum of  $I_2'$  and  $I_0$  gives the total primary current  $I_1$  which is at an angle  $\phi_1$  from  $V_1$ .

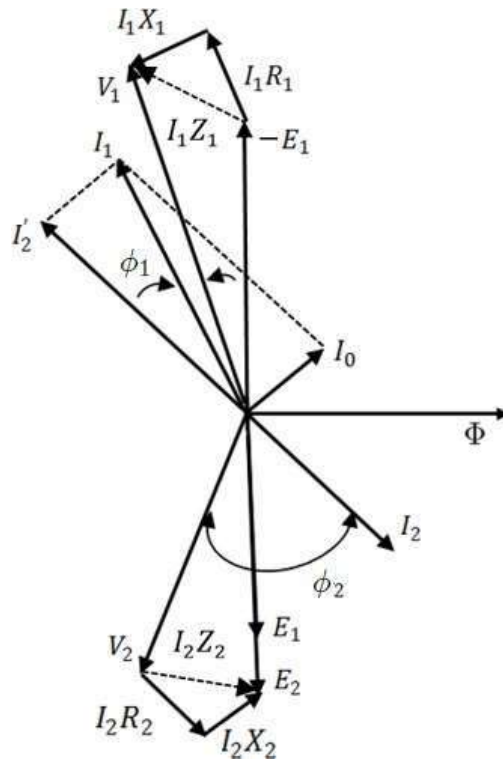


Figure 3.10: •

Now the induced e.m.f. in the primary side that opposes the applied voltage  $V_1$  is  $E_1$  and is drawn opposite to  $E_1$  as  $-E_1$ . The resistance voltage drop across the primary winding  $I_1R_1$  is drawn in parallel to the primary current  $I_1$  and the reactance voltage drop in the primary winding  $I_1X_1$  is drawn in perpendicular to the primary current  $I_1$  from  $-E_1$ . The vector sum of  $I_1R_1$  and  $I_1X_1$  gives  $I_1Z_1$ . Also the vector sum of  $-E_1$  and  $I_1Z_1$  gives the applied primary voltage  $V_1$ .

In the secondary side, the phasor  $E_2$  represents the induced e.m.f. in the secondary by the mutual flux  $\Phi$ . The secondary terminal voltage is  $V_2$ . Now The resistance voltage drop across the secondary winding  $I_2R_2$  is drawn in parallel to the secondary current  $I_2$  and the reactance voltage drop in the secondary winding  $I_2X_2$

is drawn in perpendicular to the secondary current  $I_2$  from  $V_2$ . The vector sum of  $I_2R_2$  and  $I_2X_2$  gives  $I_2Z_2$ . Also the vector sum of  $V_2$  and  $I_2Z_2$  gives the mutually induced e.m.f.  $E_2$  in the secondary side.

From the vector diagram, the primary impedance is given by,

$$Z_1 = R_1 + jX_1 \tag{3.20}$$

Similarly the secondary impedance is given by,

$$Z_2 = R_2 + jX_2 \tag{3.21}$$

In the primary side the induced e.m.f. is decreased because of the voltage drop due to resistive and reactance drop. the input voltage is given by,

$$\begin{aligned} V_1 &= -E_1 + I_1(R_1 + jX_1) \\ &= -E_1 + I_1Z_1 \end{aligned} \tag{3.22}$$

Similarly in the secondary side the terminal voltage is decreased because of the voltage drop due to resistive and reactance drop. the mutually induced e.m.f. is given by,

$$\begin{aligned} E_2 &= V_2 + I_2(R_2 + jX_2) \\ &= V_2 + I_2Z_2 \end{aligned} \tag{3.23}$$

### 3.11 Equivalent Circuit

The equivalent circuit diagram of a transformer is essential for the predetermination of the behaviour of the transformer under the various operating conditions. It is simply the representation of the transformer using resistance, inductance, capacitance, voltage, current etc. The equivalent circuit of transformer can be obtained referred to primary side or secondary side. The equivalent circuit of transformer is shown in Figure 3.11. In this circuit, the resistance and leakage reactance is shown outside of the transformer.

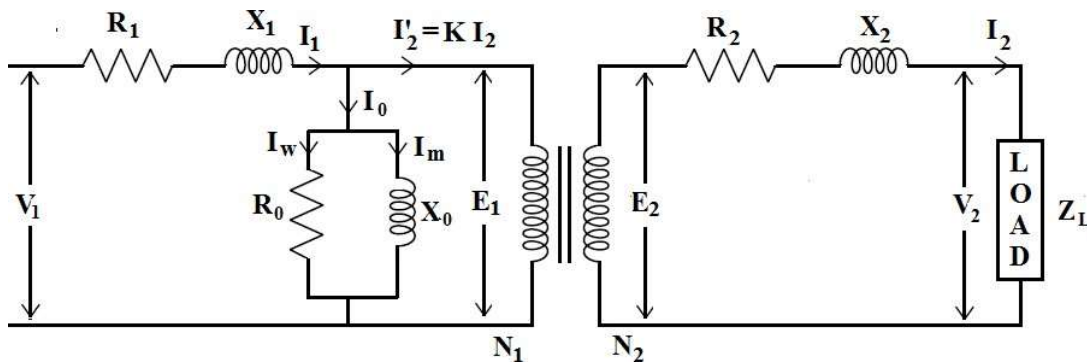


Figure 3.11: .

## Transformers

The no load current  $I_0$  is divided into, magnetising component of current  $I_m$  which is the current flowing through pure inductance  $X_0$  and working component of current  $I_w$  which is the current flowing through non inductive resistance  $R_0$ . The value of  $X_0 = \frac{E_1}{I_m}$  and  $R_0 = \frac{E_1}{I_w}$ . Also  $E_1$  and  $E_2$  are related by,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

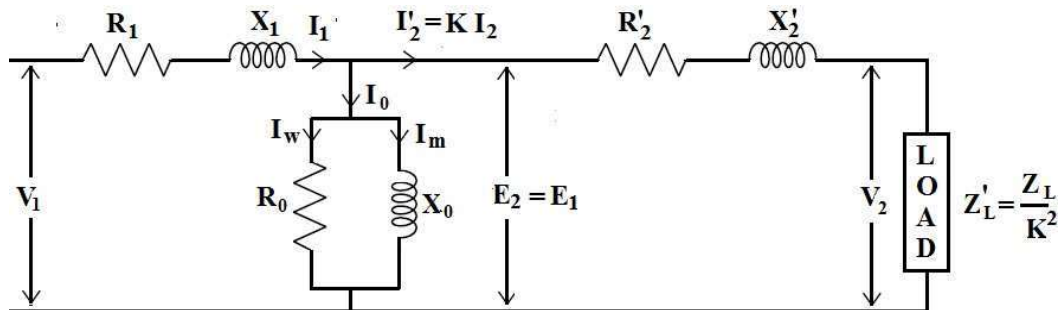


Figure 3.12: •

The equivalent circuit can be simplified by transferring the voltage, current resistance etc. to primary or secondary side.

Secondary induced voltage referred to primary,  $E_2' = \frac{E_2}{K}$

Secondary terminal voltage referred to primary,  $V_2' = \frac{V_2}{K}$

Secondary current referred to primary,  $I_2' = K I_2$

Secondary resistance referred to primary,  $R_2' = \frac{R_2}{K^2}$

Secondary leakage reactance referred to primary,  $X_2' = \frac{X_2}{K^2}$

Load impedance referred to primary,  $Z_L' = \frac{Z_L}{K^2}$

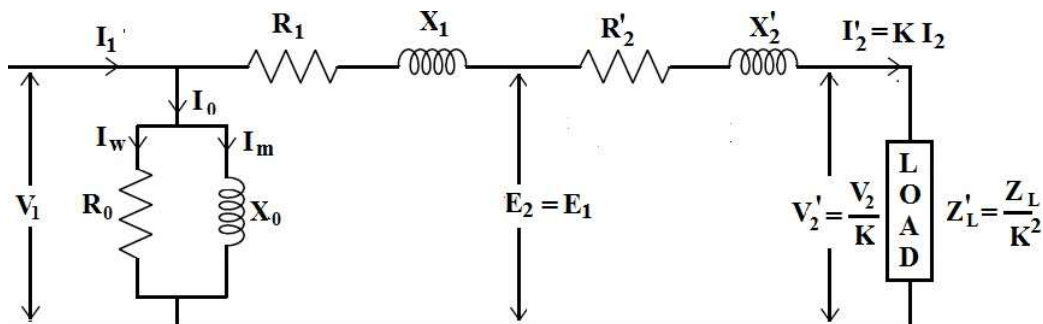


Figure 3.13: •

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to the primary as shown in Figure 3.12. This is the exact equivalent circuit of transformer.

Figure 3.13 shows the approximate equivalent circuit of the transformer. circuit of transformer with secondary parameters referred to the primary.

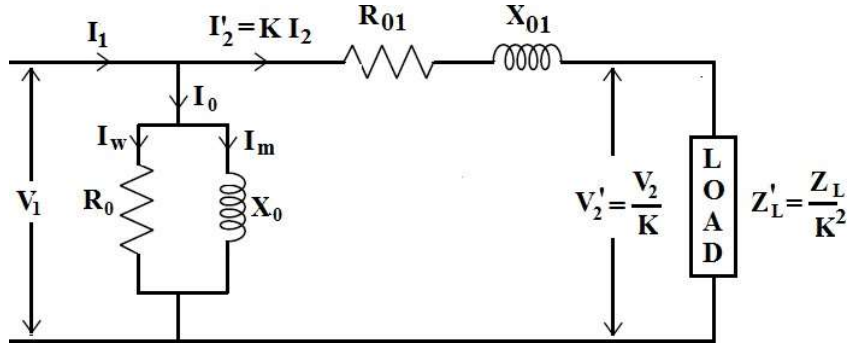


Figure 3.14: .

The equivalent circuit can be modified by transferring the resistance  $R_0$  and leakage reactance  $X_0$  towards left end. Where equivalent resistance of transformer is  $R_{01} = R_1 + R_2$  and equivalent leakage reactance  $X_{01} = X_1 + X_2$

### 3.12 Open Circuit Test

The Open circuit test is a no-load test carried out to determine the no-load loss or core loss or iron loss and the no-load current  $I_0$ . The no-load parameters  $R_0$  and  $X_0$  can easily be determined by conducting this test. Any one winding of the transformer usually the low voltage (LV) winding is connected to the the rated voltage  $V_1$  as shown in the Figure 3.15. In this case the low voltage (LV) winding is the primary winding which is connected to a watt meter (W), a voltmeter (V) and an ammeter (A).

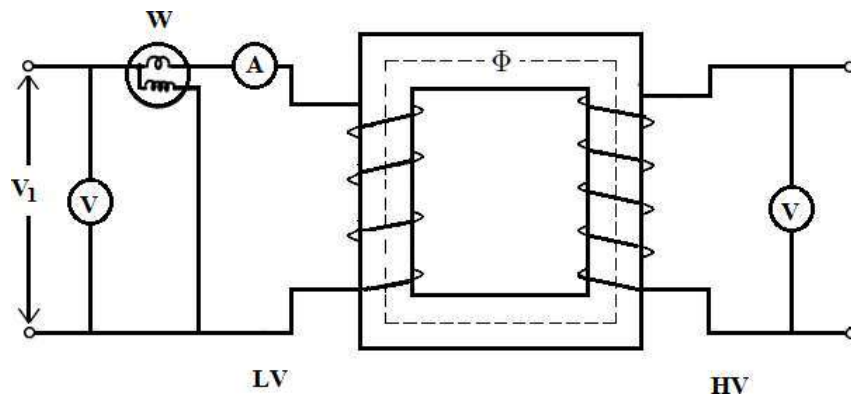


Figure 3.15: Open Circuit Test



## Transformers

The high voltage (HV) winding of the transformer is open circuited. The current drawn by the primary is no load current  $I_0$  which is measured by an ammeter connected. The value of no-load current is very small usually 2 to 10% of the rated full load current. Thus, there is no copper loss on the secondary side as it is open circuited and in the primary winding, the copper loss is negligible. Therefore, the wattmeter reading  $W$  represents the core or iron losses.

$$\text{Input power, } W = V_1 I_0 \cos \phi_0$$

$$\therefore \text{No-load power factor, } \cos \phi_0 = \frac{W}{V_1 I_0}$$

$$\text{Magnetising component of current, } I_m = I_0 \sin \phi_0$$

$$\text{Iron loss component of current, } I_w = I_0 \cos \phi_0$$

$$\text{Magnetizing reactance, } X_0 = \frac{V_1}{I_m}$$

$$\text{Iron loss resistance, } R_0 = \frac{V_1}{I_w}$$

In order to determine the transformation ratio of the transformer, a voltmeter is normally connected across the high voltage (HV) winding of the transformer. So the open circuit test can be used to determine (i) Core loss at rated voltage (ii) No load parameters  $R_0$  and  $X_0$  and (iii) Transformation ratio of the transformer.

### 3.13 Short Circuit Test

The short circuit test is a full load test used to determine the copper losses at full load. This test also helps in determine the equivalent impedance ( $Z_{01}$  or  $Z_{02}$ ), equivalent resistance ( $R_{01}$  or  $R_{02}$ ) and the equivalent leakage reactance ( $X_{01}$  or  $X_{02}$ ) of the transformer referred to the primary or secondary winding.

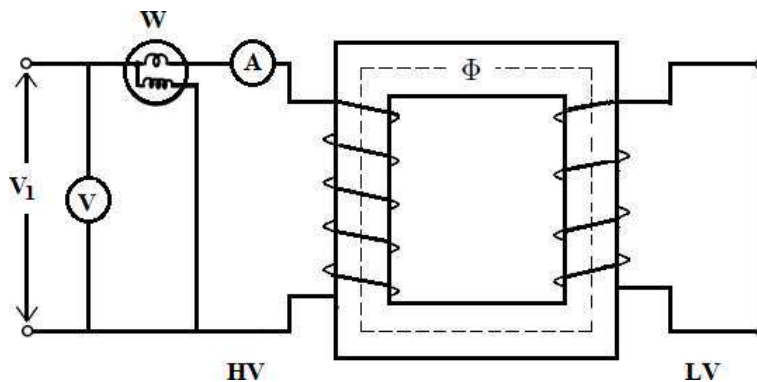


Figure 3.16: Short Circuit Test

Any one winding of the transformer usually the high voltage (HV) winding of the transformer is connected to a low voltage supply at the normal frequency, so that

full load current flows through the primary and secondary windings. The watt meter (W), a voltmeter (V) and an ammeter (A) are connected to the primary side of the transformer as shown in the above Figure 3.16. The low voltage (LV) winding or the secondary winding is short circuited. The full load current is measured by an ammeter.

The voltage applied to the primary winding is 5 to 10% of the rated voltage. Therefore, the value of the flux which is set up in the core is also small compared to the normal flux. Hence, the iron losses are negligible and the wattmeter (W) measures the full load copper loss of both primary and secondary windings.

$$\begin{aligned} \text{Full load Copper Loss, } W &= I_1^2 R_{01} \\ \therefore \text{Equivalent Resistance, } R_{01} &= \frac{W}{I_1^2} \\ \text{Equivalent Impedance, } Z_{01} &= \frac{V_1}{I_1} \\ \therefore \text{Equivalent Reactance, } X_{01} &= \sqrt{Z_{01}^2 - R_{01}^2} \end{aligned}$$

Thus the open circuit test can be used to determine (i) full load copper loss and (ii) equivalent resistance  $R_{01}$  and equivalent reactance  $X_{01}$

**Note**

Advantages of OC and SC test are

- ☞ The power required to carry out these tests is very small as compared to the full-load output of the transformer.
- ☞ These tests help us to determine the efficiency of the transformer at any load and p.f. without actually loading the transformer.
- ☞ This helps us to calculate voltage regulation of the transformer

The major disadvantages of OC and SC tests are

- ☞ In O.C. test, there is no load on the transformer while in S.C. circuit test only small percentage of rated voltage is applied. In all O.C. and S.C. tests, the loading conditions are absent. Hence the results are inaccurate
- ☞ In open and short circuit test, iron losses and copper losses are determined separately but in actual use both losses occurs simultaneously.
- ☞ The temperature rise in the transformer is due to total loss that occurs simultaneously during actual use, it can't be determined by O.C and S.C

## **Transformers**

tests.

**Example 3. 5**

Obtain the equivalent circuit of a 250/500 V, 50-Hz, single phase transformer from the following test data :

O.C test : 250 V, .8 A, 80 W – on L.V. side

S.C. test : 20 V, 15 A, 90 W – on H.V. side

**Solution : From OC test**

$$W_0 = V_1 I_0 \cos \varphi_0$$

Noload power factor,

$$\cos \varphi_0 = \frac{W_0}{V_1 I_0} = \frac{80}{250 \times .8} = 0.4$$

$$\sin \varphi_0 = \sqrt{1 - 0.4^2} = 0.92$$

$$I_w = I_0 \cos \varphi_0 = 0.8 \times 0.4 = 0.32 \text{ A}$$

$$I_m = I_0 \sin \varphi_0 = 0.8 \times 0.92 = 0.74 \text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{250}{0.32} = 781.25 \Omega$$

$$X_0 = \frac{V_1}{I_m} = \frac{250}{0.74} = 337.84 \Omega$$

**From S.C. Test**

Meters are placed in the secondary or high-voltage winding whereas the primary or low-voltage winding is short-circuited.

$$K = \frac{500}{250} = 2$$

$$Z_{02} = \frac{V_{sc}}{I_2} = \frac{20}{15} = 1.33 \Omega$$

$$Z_{01} = \frac{Z_{02}}{K} = \frac{1.33}{2} = 0.665 \Omega$$

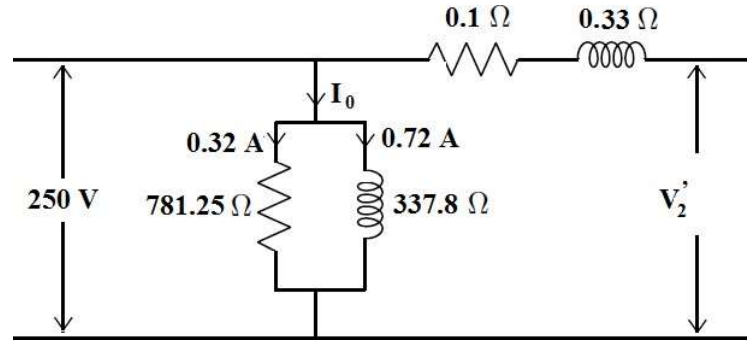
$$R_{02} = \frac{W}{I_2^2} = \frac{90}{15^2} = 0.4 \Omega$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.4}{2^2} = 0.1 \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{0.665^2 - 0.1^2} = 0.65 \Omega$$

The equivalent circuit is given below

## Transformers



### Example 3. 6

In no-load test of single-phase transformer, the following test data were obtained:  
 Primary voltage : 200 V ; Secondary voltage : 100 V ; Primary current : 0.4 A ;  
 Power input : 25 W.

If the primary winding resistance is 0.6 ohms, find the following :

(i) The turns ratio (ii) Magnetising component of no-load current (iii) its working component (iv) the iron loss. Resistance of the primary winding = 0.6 ohm.

### Solution :

Input voltage,  $V_1 = 200 \text{ V}$

primary current,  $I_0 = 0.4 \text{ A}$

Input power,  $W = V_1 I_0 \cos \phi_0$

$$\therefore \text{Noload power factor, } \cos \phi_0 = \frac{W}{V_1 I_0} = \frac{25}{200 \times 0.4} = 0.313$$

$$\sin \phi_0 = \sqrt{1 - 0.313^2} = 0.95$$

(i)

$$\text{Turns ratio, } K = \frac{V_1}{V_2} = \frac{200}{100} = 2$$

(ii) magnetising component of no-load current

$$I_m = I_0 \sin \phi_0 = 0.4 \times 0.95 = 0.38 \text{ A}$$

(iii) working component of no-load current

$$I_w = I_0 \cos \phi_0$$

$$= 0.4 \times 0.313$$

$$= 0.125 \text{ A}$$

(iv)

$$\begin{aligned} \text{Primary copper loss} &= I_0^2 R_1 \\ &= 0.4^2 \times 0.6 \\ &= 0.096 \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Iron loss} &= 25 - 0.096 \\ &= 24.9 \text{ W} \end{aligned}$$

### 3.14 Why Transformer Rating in kVA?

The copper loss of a transformer depends on current, and iron loss depends on voltage. Thus, total transformer loss depends on volt-ampere (VA). It does not depend on the phase angle between voltage and current, i.e. transformer loss is independent of load power factor. This is the reason that transformers are rated in kVA

Rating of Single Phase Transformer,

$$P = V \times I.$$

Rating of a single phase transformer in kVA,

$$\text{kVA} = \frac{V \times I}{1000}$$

Rating of a Three Phase Transformer,

$$P = \sqrt{3} \times V \times I.$$

Rating of a Three phase transformer in kVA,

$$\text{kVA} = \frac{\sqrt{3} \times V \times I}{1000}$$

#### Note

##### **Transformer on DC supply**

A transformer cannot operate on DC supply. When a DC voltage is applied to the primary of the transformer, the flux is produced in the transformer core which is constant in magnitude and not varying. So no self induced e.m.f is produced in the primary winding as it is possible only with varying flux. Also no e.m.f. will be induced in the secondary winding also.

Another thing is, the resistance of primary winding is quite small, so a large current will flow through the primary windings which may result in the burning out of primary windings.

## Transformers

### 3.15 All Day Efficiency

The efficiency of a transformer is defined as the ratio between output power to input power of the transformer at full load condition. But in the case of a distribution transformer, we cannot predict the performance by this efficiency. Distribution transformer is commonly used for domestic lighting application. The efficiency of such transformer is maximum at 50% of full load.

Iron loss does not depend upon the condition of secondary load of the transformer. In all loading condition, these are fixed. But the copper loss which is also referred as  $I^2 R$  loss entirely depends upon load .

A distribution transformer cannot be run with constant load throughout 24 hours. At peak time it's loading is high, whereas in lean time its loading may be negligible. So judging the performance of a transformer by its conventional efficiency is not practical and economical. As a solution of these problems, the concept of all day efficiency of distribution transformer came into the picture. In this concept, we use the ratio of total energy delivered by the transformer to the total energy fed to the transformer, during a 24 hours span of time.



Hence, all day efficiency is determined as, the ratio of total KWh at the secondary to the total KWh at the primary of the transformer for a long specific time period preferably 24 hours. i.e,

$$\eta_{\text{all-day}} = \frac{\text{Output in KWh (For 24 hours)}}{\text{Input in KWh}} \quad (3.24)$$

This is very much useful to judge the performance of a distribution transformer, whose primary is energised throughout the day, but secondary load varies considerably throughout the day.

#### Determination of all-day efficiency

For the determination of all-day efficiency, we need to know the load variational from hour to hour during the day. The copper loss depends upon the load but the iron loss remains constant throughout the day. The all-day efficiency of a transformer will be lesser than the ordinary or commercial efficiency of a transformer.

**Example 3. 7**

Find “all day” efficiency of a transformer having maximum efficiency of 98 at 20 kVA at unity power factor and loaded as follows :

Time in hour	Loading in KW	Power factor
10	2	0.5 lag
8	12	0.8 lag
6	at no load.	-

**Solution**

$$\text{Output} = \text{KVA} \times p.f. = 20 \times 1 = 20 \text{ kW}$$

$$\text{Input} = \frac{20}{0.98} = 20.408 \text{ KW}$$

$$\text{Losses} = 20.408 - 20 = 0.408 \text{ kW} = 408 \text{ W}$$

Since efficiency is maximum, cu loss is equal to core loss

$$\text{Iron loss} = \frac{408}{2} = 204 \text{ W}$$

$$\text{Full load Cu loss} = \frac{408}{2} = 204 \text{ W}$$

$$2 \text{ kW at } 0.5 \text{ p.f.} = \frac{2}{0.5} = 4 \text{ kVA}$$

$$12 \text{ kW at } 0.8 \text{ p.f.} = \frac{12}{0.8} = 15 \text{ kVA}$$

$$\text{Cu loss at } 4 \text{ KVA} = 204 \times \left(\frac{4}{20}\right)^2 = 8.16 \text{ W}$$

$$\text{Cu loss at } 15 \text{ KVA} = 204 \times \left(\frac{15}{20}\right)^2 = 114.75 \text{ W}$$

$$\text{Cu loss for } 10 \text{ hours} = 8.16 \times 10 = 81.6 \text{ Whr}$$

$$\text{Cu loss for } 8 \text{ hours} = 114.75 \times 8 = 918 \text{ Whr}$$

$$\text{Total Cu loss for } 24 \text{ hours} = 81.6 + 918 = 1000 \text{ Whr} = 1 \text{ KWhr}$$

$$\text{Total Iron loss for } 24 \text{ hours} = 204 \times 24 = 4896 \text{ Whr} = 4.9 \text{ KWhr}$$

$$\text{Output for } 24 \text{ hours} = (10 \times 2) + (8 \times 12) = 116 \text{ KWhr}$$

$$\text{Input for } 24 \text{ hours} = 116 + 1 + 4.9 = 121.9 \text{ KWhr}$$

$$\begin{aligned} \eta_{\text{all-day}} &= \frac{116}{121.9} \times 100 \\ &= 95.16\% \end{aligned}$$



## Transformers

### 3.16 Constant Voltage Transformer

Constant Voltage Transformer which is also known as *ferro-resonant transformer* is simply a magnetic transformer of special construction used to obtain constant voltage. It has a capacitor which is connected across the secondary winding of the transformer to tune out the output at a frequency very close to 50 Hz. It works on the principle of *ferro-resonance*. It will make the output voltage constant by the saturation of iron core. So the output voltage will be a square wave. It can instantaneously correct the voltage even under sudden dips and spikes. It is commonly used in stabilizers, Ups, personal computer, biomedical equipment, telex, cash registers, electronic typewriters etc.

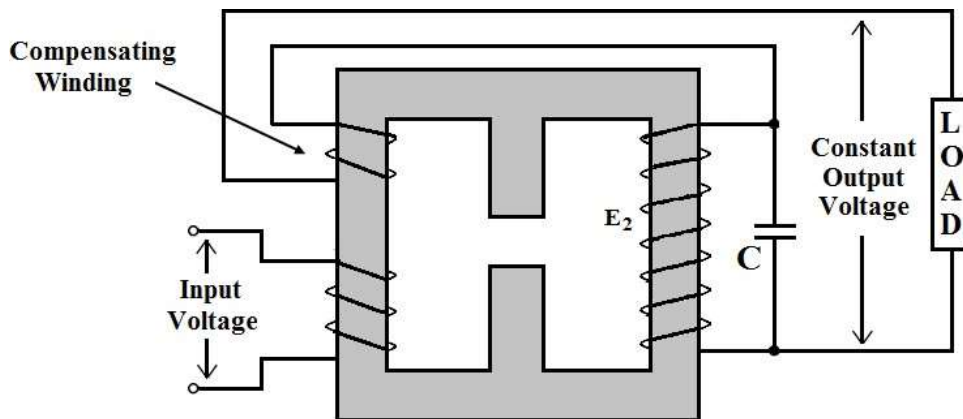


Figure 3.17: Constant Voltage Transformer

When the input voltage is low, the capacitor will provides the necessary additional power keeping the output voltage steady. This capacitor also increases the current in the secondary winding which helps in the saturation of iron core and for a large range of the input voltage, a constant voltage is available across the secondary winding. On the other hand , when mains voltage is high, to keep the output voltage steady, the leakage reactance will drop the difference of voltage.

The primary and secondary windings of CVT are wound separately from each other. A separate shunt path is provided, to set up magnetic field in between the two windings. An airgap is formed in the shunt path. A capacitor is connected across a small portion of secondary winding. The air gap is provided in the magnetic shut for providing a constant voltage output. Resonant action of the secondary winding of the transformer and the capacitor is responsible for the operation of CVT. compensating winding is used to improve the regulation of CVT.

#### 3.16.1 Advantages

1. It gives constant output voltage, regardless of current variation.

2. Faster voltage correction and overload protection.
3. It provides an excellent output noise reduction.
4. It will provide a low impedance path under extreme surge voltage such as a local lightning strike and protect both itself and any connected loads.
5. It gives constant spike free voltage with better voltage regulation.

### 3.17 Autotransformer

Autotransformer is a kind of transformer having only one winding wound on a laminated silicon steel core. A part of the winding is common to both primary and secondary sides. So in the case of an autotransformer, the primary and secondary windings are electrically and magnetically connected with each other. An Auto transformer works as a voltage regulator. Note that the primary and secondary windings are connected electrically as well as magnetically. Therefore, power from the primary is transferred to the secondary conductively as well as inductively.

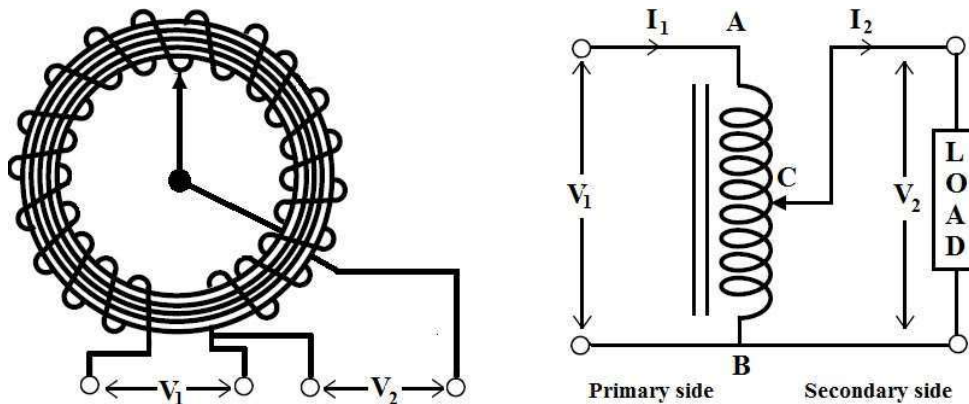


Figure 3.18: Auto Transformer

Figure 3.18 shows the schematic diagram of an auto transformer in which AB is the primary winding having  $N_1$  turns and BC is the secondary winding having  $N_2$  turns. The transformation ratio of autotransformer is given by,

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K \quad (3.25)$$

#### 3.17.1 Saving of Copper in Autotransformer

The length of the conductor is proportional to the number of turns of the winding and the area of cross section of the conductor depends on the current to be carried out. Hence the weight of the copper is proportional to the length and area of a cross section of the conductor. Also section AB has  $N_1$  turns and the section CB has  $N_2$  turns

## Transformers

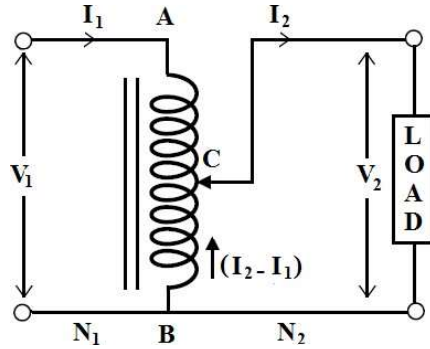


Figure 3.19: •

From the Figure 3.19, we have,

$$\text{Weight of copper in section AC} \propto (N_1 - N_2)I_1$$

$$\text{Weight of copper in section CB} \propto N_2(I_2 - I_1)$$

$$\therefore \text{Weight of copper in autotransformer} \propto (N_1 - N_2)I_1 + N_2(I_2 - I_1)$$

Ordinary transformer has separate primary and secondary windings

$$\text{Weight of copper in the primary} \propto N_1I_1$$

$$\text{Weight of copper in the secondary} \propto N_2I_2$$

$$\therefore \text{Weight of copper in ordinary transformer} \propto N_1I_1 + N_2I_2$$

$$\begin{aligned} \frac{\text{Weight of copper in autotransformer, } W_a}{\text{Weight of copper in ordinary transformer, } W_o} &= \frac{(N_1 - N_2)I_1 + N_2(I_2 - I_1)}{N_1I_1 + N_2I_2} \\ &= \frac{N_1I_1 + N_2I_2 - 2N_2I_1}{N_1I_1 + N_2I_2} \\ &= 1 - \frac{2N_2I_1}{N_1I_1 + N_2I_2} \end{aligned}$$

Divide the Nr. and Dr. of the second term by  $N_1I_1$

$$\begin{aligned} &= 1 - \frac{2N_2I_1}{N_1I_1 + N_2I_2} \\ &= 1 - \frac{2 \times \frac{N_2}{N_1}}{1 + \frac{N_2}{N_1} \times \frac{I_2}{I_1}} \\ &= 1 - \frac{2 \times K}{1 + K \times \frac{1}{K}} \\ &= 1 - K \end{aligned}$$

Where  $\frac{N_2}{N_1} = K$  and  $\frac{I_2}{I_1} = \frac{1}{K}$

$$\therefore W_a = (1 - K) \times W_0 \quad (3.26)$$

$$\text{Saving of copper} = W_0 - W_a \quad (3.27)$$

$$= W_0 - (1 - K) \times W_0 \quad (3.28)$$

$$= K W_0 \quad (3.29)$$

$$\therefore \text{Saving of copper} = K \times \text{Weight of copper in ordinary transformer} \quad (3.30)$$

### Note

In an auto transformer,

Power transformed inductively =  $(1 - K) \times$  Input power

Power transformed conductively =  $(K) \times$  Input power

### 3.17.2 Advantages of Autotransformer

1. Less costly compared to two winding transformer of same rating.
2. Saving in core material as well as conductor material.
3. Better voltage regulation due to less leakage reactance
4. Low losses as compared to ordinary two winding transformer of the same rating
5. Higher efficiency than two winding transformer due to low losses.
6. It requires smaller exciting current than a two-winding transformer of the same rating.

### 3.17.3 Disadvantages of Autotransformer

1. As there is an electrical connection between the primary and secondary, if the neutral point is not properly earthed, the lower voltage side can be subjected to high potential when there is an earth fault on high voltage side.
2. High standard of insulation is required compared to two winding transformer.
3. Surge Arrester protection is required
4. As the short circuit impedance values of the auto-transformer are low, there is a possibility of higher short circuit current.

## Transformers

### 3.17.4 Application of Autotransformer

1. Autotransformers are used as boosters to compensate for voltage drops in transmission and distribution lines.
2. Autotransformers are used as a starters for AC motors.
3. Autotransformers are used for continuously variable supply.

### 3.18 Instrument Transformer

Instrument Transformers are used in AC system for the measurement of large voltage and current. The voltage and current level of power system is very high. It is very difficult and costly to design the measuring instruments for measurement of such high level voltage and current. Basic function of Instrument transformers is to step down the AC System voltage and current, so that they can be measured easily. Generally measuring instruments are designed for 5 A and 110 V. Instrument transformers also protect the operator, the measuring devices, and the control equipment from the dangers of high voltage. The use of instrument transformers results in increased safety, accuracy, and convenience. Instrument transformers are of two types, (i) Potential Transformer and (ii) Current Transformer.

The *Burden* of an Instrument Transformer is the rated Volt-Ampere loading connected to its secondary winding which is permissible without errors exceeding the limits for a particular class of Instrument Transformer.

#### 3.18.1 Potential Transformer

Working principle of potential transformer or voltage transformer is same as that of a power transformer. Potential transformer is used in the electrical power system for stepping down the system voltage to a safe value so that it can be connected to low rating meters and relays.

Potential transformer has a primary coil of large number of turns of fine wire connected in parallel with the line whose voltage is to be measured. The secondary consists of one or more turns of thick wire. The capacity of a potential transformer is small compared to that of power transformers. Potential transformers have ratings from 100 to 500 volt-amperes (VA). The secondary side is usually wound for 115 volts and it is usually connected to low voltage meters.



In an ideal potential transformer or voltage transformer, when rated burden gets

connected across the secondary, the ratio of primary and secondary voltages of transformer is equal to the turns ratio

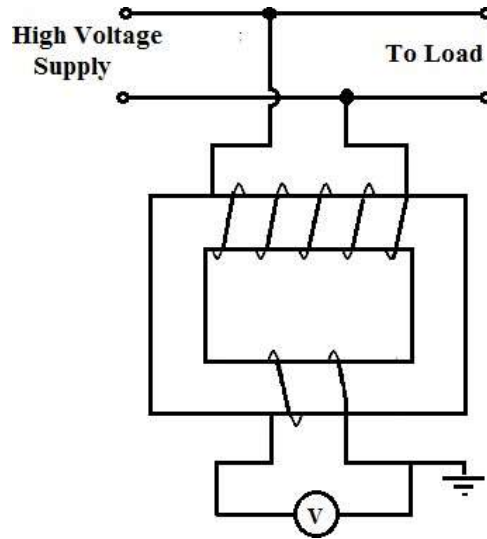


Figure 3.20: Measurement of Voltage using Potential Transformer

Consider a potential transformer is connected to a single phase line having a voltage of 2300 V. A voltmeter connected across the secondary of the potential transformer indicates a value of 115 volts. To determine the actual voltage on the high-voltage circuit, the instrument reading of 115 volts must be multiplied by 20 that is  $115 \times 20 = 2300$  volts. Where  $\frac{2300}{115} = 20$  is the voltage ratio  $\frac{V_1}{V_2}$ . Thus voltage transformer reduced the voltage to  $1/20^{th}$  of it's actual value.

### 3.18.2 Current Transformer

Current transformer is used in the electrical power system for stepping down the high alternating current to a safe value so that it can be connected to low range ammeters.

In addition to step down the current in a known ratio, the current transformer provides insulation to the instrument from the high voltage line. Current transformer has a primary coil of one or more turns of thick wire connected in series with the line whose current is to be measured. The secondary winding of the current transformer consists of large number of coil turns wound on a laminated core.



The cross sectional area of the core depending upon how much the current to be stepped down.

## Transformers

Consider a current transformer is connected to a single phase line having a current of 100 A flowing through it. An ammeter connected in the secondary of the current transformer indicates a value of 5 A. To determine the actual voltage on the high-voltage circuit, the instrument reading of 5 A must be multiplied by 20 that is  $5 \times 20 = 100$  A. Where  $\frac{100}{5} = 20$  is the current ratio  $\frac{I_1}{I_2}$ . Thus current transformer reduced the current to  $1/20^{th}$  of it's actual value.

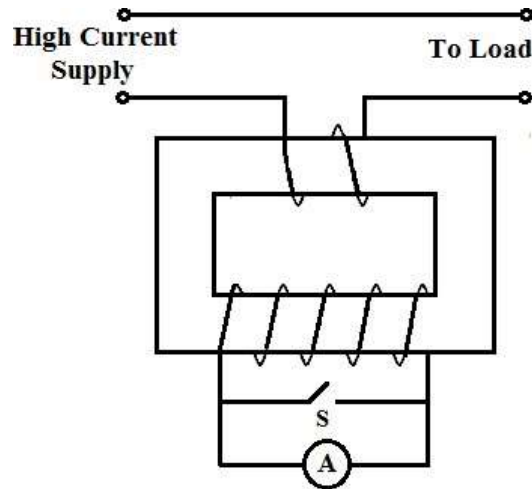


Figure 3.21: Measurement of Current using Current Transformer

It should be noted that, since the ammeter resistance is very low, the current transformer normally works like it is short circuited. If the secondary of current transformer is open circuited or the ammeter is taken out of the secondary winding, the secondary must be short circuited using switch S. If the secondary of the current transformer is open circuited, then there is no secondary ampere-turns or counter ampere-turns. The primary m.m.f. will set up an abnormally high flux in the core due to the absence of counter m.m.f by the secondary. The primary m.m.f. will produce excessive core loss with subsequent heating and a high voltage across the secondary terminals. Hence, the secondary of a current transformer should never be left open under any circumstances.

### 3.19 Voltage Regulation of Transformer

When the transformer is loaded, the secondary voltage reduces with increase in load. Voltage regulation transformer is expressed as the change in voltage from a no-load condition to full-load condition, and is expressed as a percentage of no-load. It is expressed in the following formula

$$\text{Voltage regulation} = \frac{V_{02} - V_2}{V_{02}} \quad (3.31)$$

$$\text{Percentage Voltage regulation} = \frac{V_{02} - V_2}{V_{02}} \times 100 \quad (3.32)$$

where  $V_{02}$  is the no-load secondary voltage and  $V_2$  is the full-load secondary voltage of the transformer. Performance of the transformer can be expressed in terms of voltage regulation. Lower the voltage regulation better is the transformer. It may be noted that the voltage regulation of the transformer will be the same whether primary or secondary side is considered.

For a load having a leading p.f.  $\cos \phi$

$$\text{Voltage regulation} = \frac{I_2 R_{02} \cos \phi - I_2 X_{02} \sin \phi}{V_{02}} \quad (3.33)$$

For a load having a lagging p.f.  $\cos \phi$

$$\text{Voltage regulation} = \frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{V_{02}} \quad (3.34)$$

**Note**

If the circuit is referred to primary, then it can be easily established that: For a load having a leading p.f.  $\cos \phi$

$$\text{Voltage regulation} = \frac{I_1 R_{01} \cos \phi - I_1 X_{01} \sin \phi}{V_{02}} \quad (3.35)$$

For a load having a lagging p.f.  $\cos \phi$

$$\text{Voltage regulation} = \frac{I_1 R_{01} \cos \phi + I_1 X_{01} \sin \phi}{V_{02}} \quad (3.36)$$

**3.19.1 Condition for Maximum Regulation**

For maximum efficiency

$$\begin{aligned} \frac{d}{d\phi} (\text{Voltage regulation}) &= 0 \\ \frac{d}{d\phi} \frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{V_{02}} &= 0 \\ -\frac{I_2 R_{02}}{V_{02}} \sin \phi + \frac{I_2 X_{02}}{V_{02}} \cos \phi &= 0 \\ \frac{I_2 X_{02}}{V_{02}} \cos \phi &= \frac{I_2 R_{02}}{V_{02}} \sin \phi \\ \therefore \tan \phi &= \frac{X_{02}}{R_{02}} \end{aligned}$$

$\therefore$  For maximum voltage regulation

$$\boxed{\phi = \tan^{-1} \frac{X_{02}}{R_{02}}} \quad (3.37)$$



## Transformers

### 3.19.2 Condition for Zero Regulation

For zero voltage regulation

$$I_2 R_{02} \cos \varphi + I_2 X_{02} \sin \varphi = 0$$

$$I_2 X_{02} \sin \varphi = -I_2 R_{02} \cos \varphi \tan \varphi = \frac{-R_{02}}{X_{02}}$$

∴ For zero voltage regulation

$$\varphi = \tan^{-1} \frac{-R_{02}}{X_{02}} \quad (3.38)$$

From Figure 3.22 the voltage regulation is zero at leading power factor

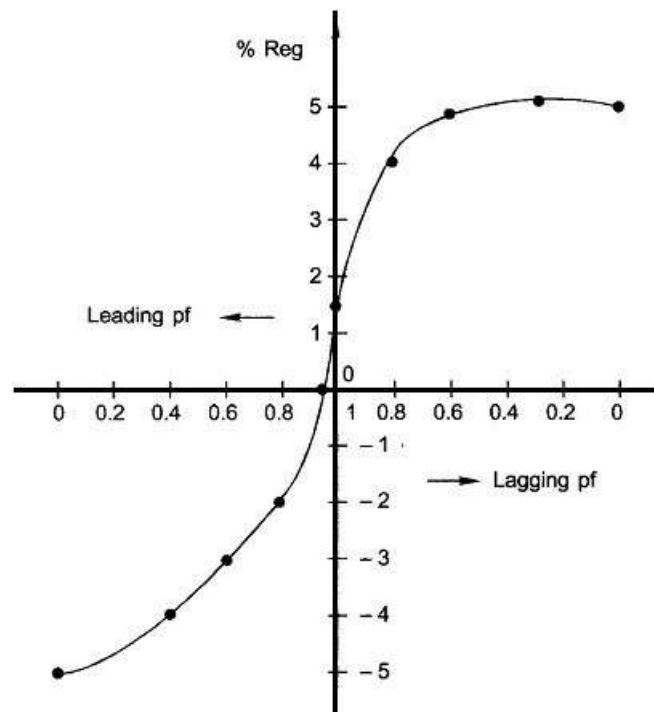


Figure 3.22: Variation of percentage regulation with power factor

Load KVA corresponding to maximum efficiency is given by

$$\text{KVA for maximum efficiency} = \frac{\text{S}}{\text{Full load KVA}} = \frac{\text{Iron loss}}{\text{Full load copper loss}}$$

## MODULE 4

*Three phase induction motors- slip ring and squirrel cage types- principles of operation – rotating magnetic field- torque slip characteristics- no load and blocked rotor tests. Circle diagrams- methods of starting – direct online – auto transformer starting*

### Three Phase Induction Motors

The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control.

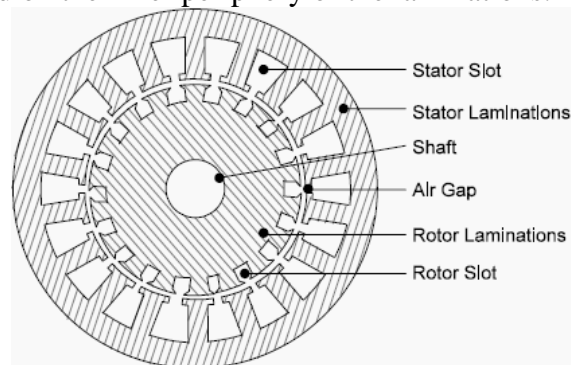
Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a “transformertype” a.c. machine in which electrical energy is converted into mechanical energy.

#### Construction

A 3-phase induction motor has two main parts (i) stator and (ii) rotor. The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.

#### Stator

It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations.



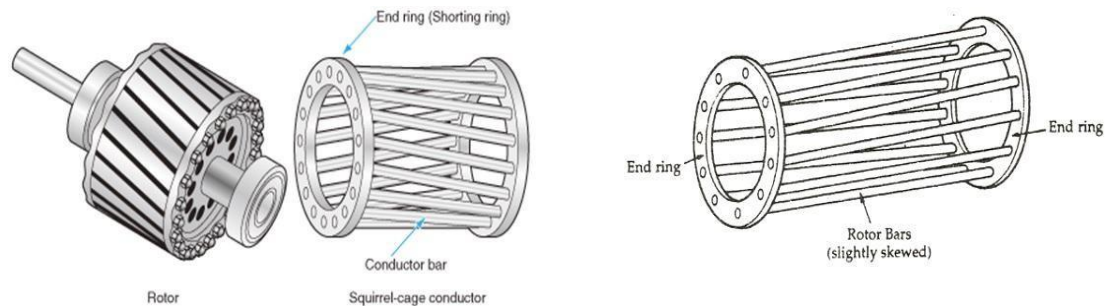
The insulated windings connected to form a balanced 3-phase star or delta connected circuit. The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.

#### Rotor

The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

- (i) Squirrel cage type
- (ii) Wound type

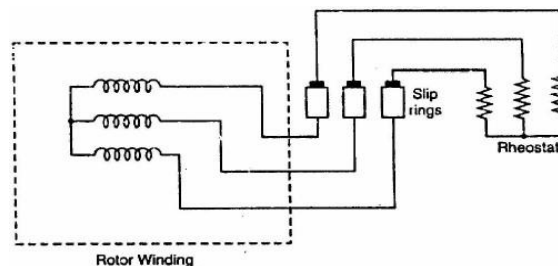
(i) **Squirrel cage rotor.** It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminium bar is placed in each slot. All these bars are joined at each end by metal rings called end rings.



This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator.

Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.

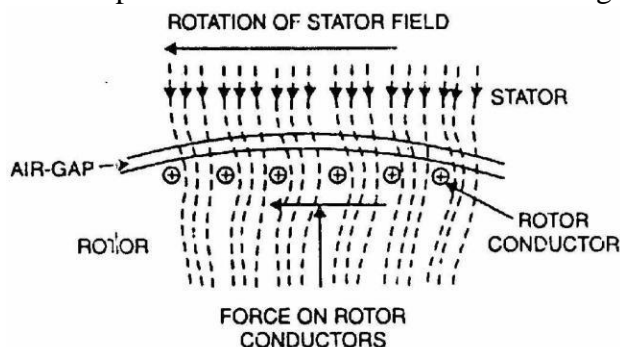
(ii) **Wound rotor.** It consists of a laminated cylindrical core and carries a 3-phase winding, similar to the one on the stator. The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The three brushes are connected to a 3-phase star-connected rheostat as shown in Fig.



At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed. The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

### Principle of Operation of 3 phase IM

Consider a portion of 3-phase induction motor as shown in Fig.



- (i) When 3-phase stator winding is energized from a 3-phase supply, a rotating magnetic field is set up which rotates round the stator at synchronous speed  $N_s (= 120 f/P)$ .
- (ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary. Due to the relative speed between the rotating flux and the stationary rotor, e.m.f.s are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors.
- (iii) The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field.
- (iv) The fact that rotor is urged to follow the stator field (i.e., rotor moves in the direction of stator field) can be explained by Lenz's law. According to this law, the direction of rotor currents will be such that they tend to oppose the cause producing them. Now, the cause producing the rotor currents is the relative speed between the rotating field and the stationary rotor conductors. Hence to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it.

### Slip

In Induction motor the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed ( $N$ ) is always less than the stator field speed ( $N_s$ ). This difference in speed depends upon load on the motor.

The difference between the synchronous speed  $N_s$  of the rotating stator field and the actual rotor speed  $N$  is called slip. It is usually expressed as a percentage of synchronous speed i.e.,

$$\% \text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

- (i) The quantity  $N_s - N$  is sometimes called slip speed.
- (ii) When the rotor is stationary (i.e.,  $N = 0$ ), slip,  $s = 1$  or 100 %.
- (iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

### Rotor Torque

The torque  $T$  developed by the rotor is directly proportional to:

- (i) rotor current
- (ii) rotor e.m.f.
- (iii) power factor of the rotor circuit

$$\begin{aligned} \therefore T &\propto E_2 I_2 \cos \phi_2 \\ \text{or } T &= K E_2 I_2 \cos \phi_2 \\ &\text{where } I_2 = \text{rotor current at standstill} \\ &E_2 = \text{rotor e.m.f. at standstill} \\ &\cos \phi_2 = \text{rotor p.f. at standstill} \end{aligned}$$

**Note.** The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

### Starting Torque ( $T_s$ )

Let  $E_2$  = rotor e.m.f. per phase at standstill

$X_2$  = rotor reactance per phase at standstill

$R_2$  = rotor resistance per phase

$$\text{Rotor impedance/phase, } Z_2 = \sqrt{R_2^2 + X_2^2} \quad \dots \text{at standstill}$$

$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \quad \dots \text{at standstill}$$

$$\text{Rotor p.f., } \cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \quad \dots \text{at standstill}$$

$$\begin{aligned} \therefore \text{ Starting torque, } T_s &= K E_2 I_2 \cos \phi_2 \\ &= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}} \\ &= \frac{K E_2^2 R_2}{R_2^2 + X_2^2} \end{aligned}$$

Generally, the stator supply voltage  $V$  is constant so that flux per pole  $\phi$  set up by the stator is also fixed. This in turn means that e.m.f.  $E_2$  induced in the rotor will be constant.

$$\therefore T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where  $K_1$  is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of  $R_2$  and  $X_2$  i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that  $K = 3/2 \pi N_s$ .

$$\therefore T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here  $N_s$  is in r.p.s.

### Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal to standstill rotor reactance/phase.

$$\text{Now } T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \quad (i)$$

Differentiating eq. (i) w.r.t.  $R_2$  and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1 \left[ \frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

$$\text{or } R_2^2 + X_2^2 = 2R_2^2$$

$$\text{or } R_2 = X_2$$

Hence starting torque will be maximum when:

Rotor resistance/phase = Standstill rotor reactance/phase

Under the condition of maximum starting torque,  $\phi_2 = 45^\circ$  and rotor power factor is 0.707 lagging

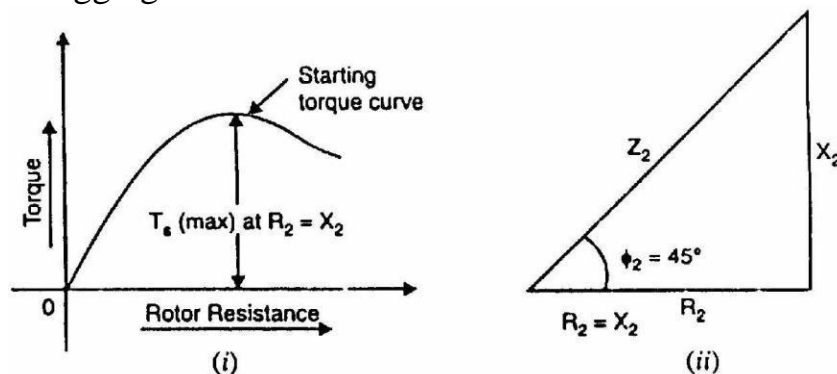


Fig. shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when  $R_2 = X_2$ . If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

### Torque Under Running Conditions

Let the rotor at standstill have per phase induced e.m.f.  $E_2$ , reactance  $X_2$  and resistance  $R_2$ . Then under running conditions at slip  $s$ ,

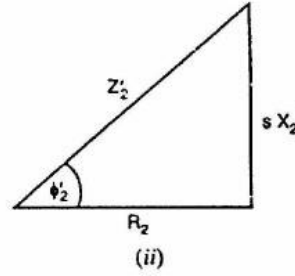
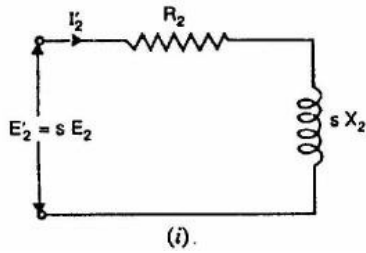
Rotor e.m.f./phase,  $E'_2 = sE_2$

Rotor reactance/phase,  $X'_2 = sX_2$

Rotor impedance/phase,  $Z'_2 = \sqrt{R_2^2 + (sX_2)^2}$

Rotor current/phase,  $I'_2 = \frac{E'_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$

Rotor p.f.,  $\cos \phi'_m = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$



Running Torque,  $T_r \propto E'_2 I'_2 \cos \phi'_2$

$$\propto \phi I'_2 \cos \phi'_2$$

$$(\because E'_2 \propto \phi)$$

$$\propto \phi \times \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (s X_2)^2}}$$

$$\propto \frac{\phi s E_2 R_2}{R_2^2 + (s X_2)^2}$$

$$= \frac{K \phi s E_2 R_2}{R_2^2 + (s X_2)^2}$$

$$= \frac{K_1 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$(\because E_2 \propto \phi)$$

If the stator supply voltage  $V$  is constant, then stator flux and hence  $E_2$  will be constant.

$$\therefore T_r = \frac{K_2 s R_2}{R_2^2 + (s X_2)^2}$$

where  $K_2$  is another constant.

It may be seen that running torque is:

- (i) directly proportional to slip i.e., if slip increases (i.e., motor speed decreases), the torque will increase and vice-versa.
- (ii) directly proportional to square of supply voltage ( $\because E_2 \propto V$ ).

It can be shown that value of  $K_1 = 3/2 \pi N_s$  where  $N_s$  is in r.p.s.

$$\therefore T_r = \frac{3}{2\pi N_s} \cdot \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} = \frac{3}{2\pi N_s} \cdot \frac{s E_2^2 R_2}{(Z'_2)^2}$$

At starting,  $s = 1$  so that starting torque is

$$T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

### Maximum Torque under Running Conditions

$$T_r = \frac{K_2 s R_2}{R_2^2 + s^2 X_2^2} \quad (i)$$

In order to find the value of rotor resistance that gives maximum torque under running conditions, differentiate exp. (i) w.r.t.  $s$  and equate the result to zero i.e.,

$$\frac{dT_r}{ds} = \frac{K_2 [R_2 (R_2^2 + s^2 X_2^2) - 2s X_2^2 (s R_2)]}{(R_2^2 + s^2 X_2^2)^2} = 0$$

$$\text{or} \quad (R_2^2 + s^2 X_2^2) - 2s X_2^2 = 0$$

$$\text{or} \quad R_2^2 = s^2 X_2^2$$

$$\text{or} \quad R_2 = s X_2$$

Thus for maximum torque ( $T_m$ ) under running conditions :

Rotor resistance/phase = Fractional slip  $\times$  Standstill rotor reactance/phase

$$\text{Now} \quad T_r \propto \frac{s R_2}{R_2^2 + s^2 X_2^2} \quad \dots \text{ from exp. (i) above}$$

For maximum torque,  $R_2 = s X_2$ . Putting  $R_2 = s X_2$  in the above expression, the maximum torque  $T_m$  is given by;

$$T_m \propto \frac{1}{2 X_2}$$

Slip corresponding to maximum torque,  $s = R_2/X_2$ .

It can be shown that:

$$T_m = \frac{3}{2\pi N_s} \cdot \frac{E_2^2}{2 X_2} \text{ N - m}$$



It is evident from the above equations that:

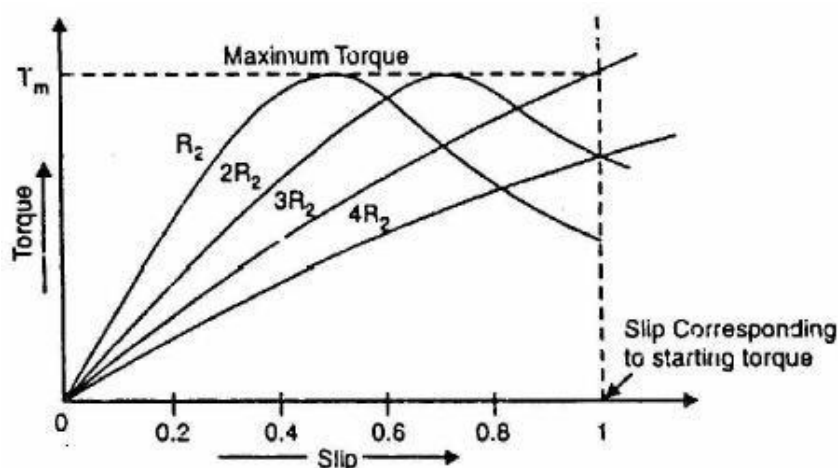
- (i) The value of rotor resistance does not alter the value of the maximum torque but only the value of the slip at which it occurs.
- (ii) The maximum torque varies inversely as the standstill reactance. Therefore, it should be kept as small as possible.
- (iii) The maximum torque varies directly with the square of the applied voltage.
- (iv) To obtain maximum torque at starting ( $s = 1$ ), the rotor resistance must be made equal to rotor reactance at standstill.

### Torque-Slip Characteristics

The motor torque under running conditions is given by;

$$T = \frac{K_2 s R_2}{R_2^2 + s^2 X_2^2}$$

If a curve is drawn between the torque and slip for a particular value of rotor resistance  $R_2$ , the graph thus obtained is called torque-slip characteristic. Fig. shows a family of torque-slip characteristics for a slip-range from  $s = 0$  to  $s = 1$  for various values of rotor resistance.



The following points may be noted carefully:

- (i) At  $s = 0$ ,  $T = 0$  so that torque-slip curve starts from the origin.
- (ii) At normal speed, slip is small so that  $s X_2$  is negligible as compared to  $R_2$ .

$$\therefore T \propto s / R_2$$

$$\propto s \dots \text{as } R_2 \text{ is constant}$$

Hence torque slip curve is a straight line from zero slip to a slip that corresponds to full-load.

- (iii) As slip increases beyond full-load slip, the torque increases and becomes maximum at  $s = R_2/X_2$ . This maximum torque in an induction motor is called pull-out torque or break-down torque. Its value is at least twice the full-load value when the motor is operated at rated voltage and frequency.

(iv) When slip increases beyond that corresponding maximum torque, the term  $s^2 X_2^2$  increases very rapidly so that  $R_2^2$  may be neglected as compared to  $s^2 X_2^2$ .

$$\begin{aligned} \therefore T &\propto s / s^2 X_2^2 \\ &\propto 1/s \dots \quad \text{as } X_2^2 \text{ is constant} \end{aligned}$$

Thus the torque is now inversely proportional to slip. Hence torque-slip curve is a rectangular hyperbola.

(v) The maximum torque remains the same and is independent of the value of rotor resistance. Therefore, the addition of resistance to the rotor circuit does not change the value of maximum torque but it only changes the value of slip at which maximum torque occurs.

### Methods of Starting 3-Phase Induction Motors

The method to be employed in starting a given induction motor depends upon the size of the motor and the type of the motor. The common methods used to start induction motors are:

- (i) Direct-on-line starting
- (ii) Stator resistance starting
- (iii) Autotransformer starting
- (iv) Star-delta starting
- (v) Rotor resistance starting

Methods (i) to (iv) are applicable to both squirrel-cage and slip ring motors. However, method (v) is applicable only to slip ring motors. In practice, any one of the first four methods is used for starting squirrel cage motors, depending upon the size of the motor. But slip ring motors are invariably started by rotor resistance starting.

#### Methods of Starting Squirrel-Cage Motors

Except direct-on-line starting, all other methods of starting squirrel-cage motors employ reduced voltage across motor terminals at starting.

#### (i) Direct-on-line starting

This method of starting is just what the name implies—the motor is started by connecting it directly to 3-phase supply. The impedance of the motor at standstill is relatively low and when it is directly connected to the supply system, the starting current will be high (4 to 10 times the full-load current) and at a low power factor. Consequently, this method of starting is suitable for relatively small (up to 7.5 kW) machines.

#### Relation between starting and F.L. torques.

We know that:

$$\text{Rotor input} = 2\pi N_s T = kT$$

But Rotor Cu loss =  $s \times$  Rotor input

$$3(I_2')^2 R_2 = s kT$$

$$\text{or} \quad T \propto (I_2')^2 / s$$

or  $T \propto I_1^2/s$  ( $\because I_2 \propto I_1$ )

If  $I_{st}$  is the starting current, then starting torque ( $T_{st}$ ) is

$$T \propto I_{st}^2 \quad (\because \text{at starting } s=1)$$

If  $I_f$  is the full-load current and  $s_f$  is the full-load slip, then,

$$T_f \propto I_f^2/s_f$$

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f$$

When the motor is started direct-on-line, the starting current is the short-circuit (blocked-rotor) current  $I_{sc}$ .

$$\therefore \frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

Let us illustrate the above relation with a numerical example. Suppose  $I_{sc} = 5 I_f$  and full-load slip  $s_f = 0.04$ . Then,

$$\frac{T_{st}}{T_f} = \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f = \left(\frac{5 I_f}{I_f}\right)^2 \times 0.04 = (5)^2 \times 0.04 = 1$$

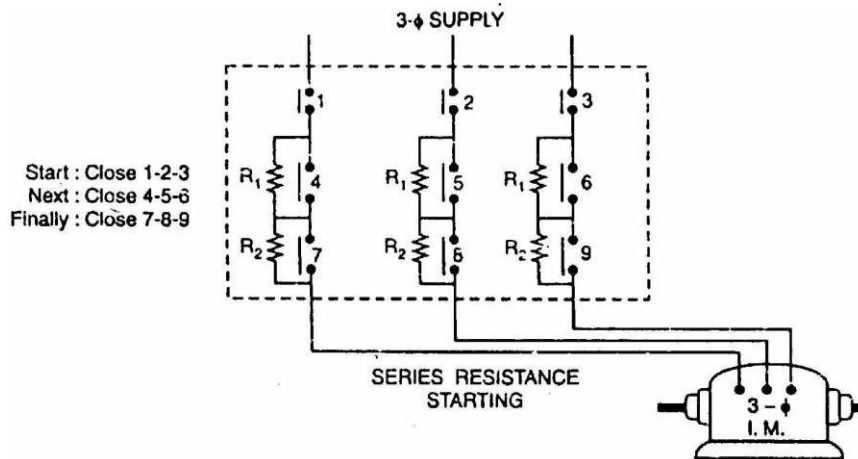
$$\therefore T_{st} = T_f$$

Note that starting current is as large as five times the full-load current but starting torque is just equal to the full-load torque. Therefore, starting current is very high and the starting torque is comparatively low. If this large starting current flows for a long time, it may overheat the motor and damage the insulation.

### (ii) Stator resistance starting

In this method, external resistances are connected in series with each phase of stator winding during starting. This causes voltage drop across the resistances so that voltage available across motor terminals is reduced and hence the starting current. The starting resistances are gradually cut out in steps (two or more steps) from the stator circuit as the motor picks up speed. When the motor attains rated speed, the resistances are completely cut out and full line voltage is applied to the rotor.

This method suffers from two drawbacks. First, the reduced voltage applied to the motor during the starting period lowers the starting torque and hence increases the accelerating time. Secondly, a lot of power is wasted in the starting resistances.



**Relation between starting and F.L. torques.** Let  $V$  be the rated voltage/phase. If the voltage is reduced by a fraction  $x$  by the insertion of resistors in the line, then voltage applied to the motor per phase will be  $xV$ .

$$I_{st} = x I_{sc}$$

Now 
$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times s_f$$

or 
$$\frac{T_{st}}{T_f} = x^2 \left(\frac{I_{sc}}{I_f}\right)^2 \times s_f$$

Thus while the starting current reduces by a fraction „ $x$ ” of the rated-voltage starting current ( $I_{sc}$ ), the starting torque is reduced by a fraction „ $x^2$ ” of that obtained by direct switching. The reduced voltage applied to the motor during the starting period lowers the starting current but at the same time increases the accelerating time because of the reduced value of the starting torque. Therefore, this method is used for starting small motors only.

**(iii) Autotransformer starting**

This method also aims at connecting the induction motor to a reduced supply at starting and then connecting it to the full voltage as the motor picks up sufficient speed.

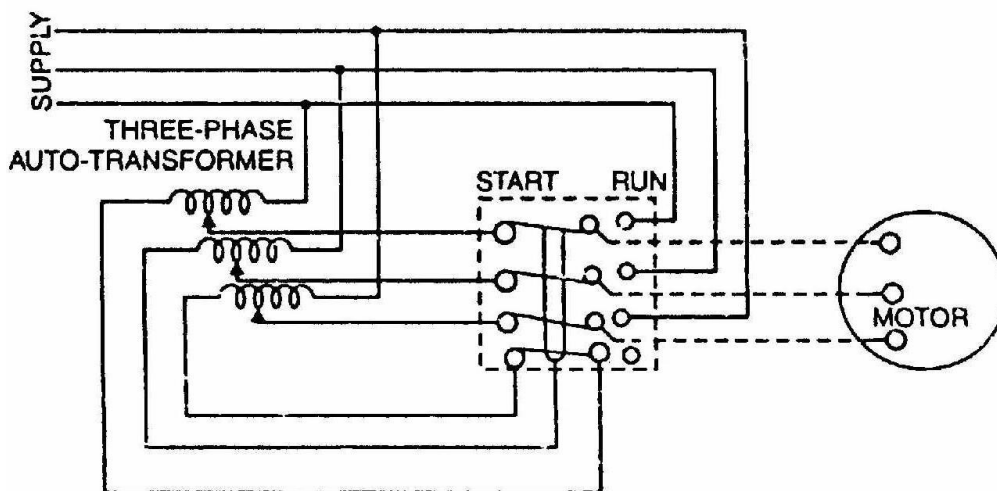


Fig. shows the circuit arrangement for autotransformer starting. The tapping on the autotransformer is so set that when it is in the circuit, 65% to 80% of line voltage is applied to the motor.

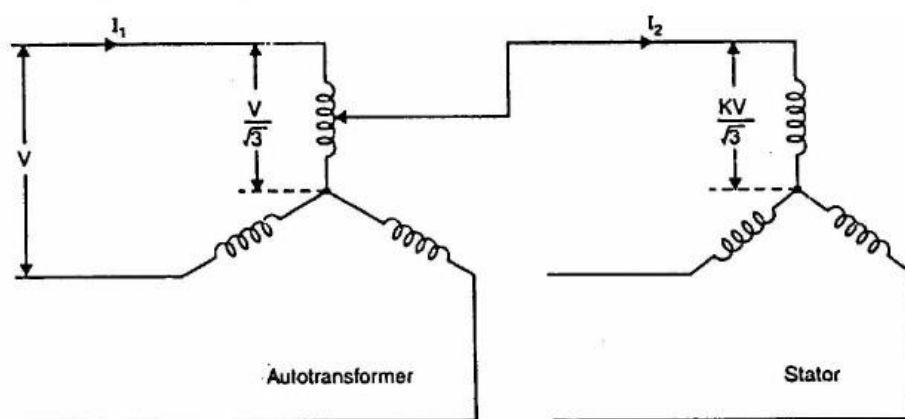
At the instant of starting, the change-over switch is thrown to “start” position. This puts the autotransformer in the circuit and thus reduced voltage is applied to the circuit. Consequently, starting current is limited to safe value.

When the motor attains about 80% of normal speed, the changeover switch is thrown to “run” position. This takes out the autotransformer from the circuit and puts the motor to full line voltage. Autotransformer starting has several advantages viz low power loss, low starting current and less radiated heat. For large machines (over 25 H.P.), this method of starting is often used. This method can be used for both star and delta connected motors.

**Relation between starting And F.L. torques.** Consider a star-connected squirrel-cage induction motor. If  $V$  is the line voltage, then voltage across motor phase on direct switching is  $V/\sqrt{3}$  and starting current is  $I_{st} = I_{sc}$ . In case of autotransformer, if a tapping of transformation ratio  $K$  (a fraction) is used, then phase voltage across motor is  $KV/\sqrt{3}$  and  $I_{st} = K I_{sc}$ ,

$$\text{Now} \quad \frac{T_{st}}{T_f} = \left( \frac{I_{st}}{I_f} \right)^2 \times s_f = \left( \frac{K I_{sc}}{I_f} \right)^2 \times s_f = K^2 \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f$$

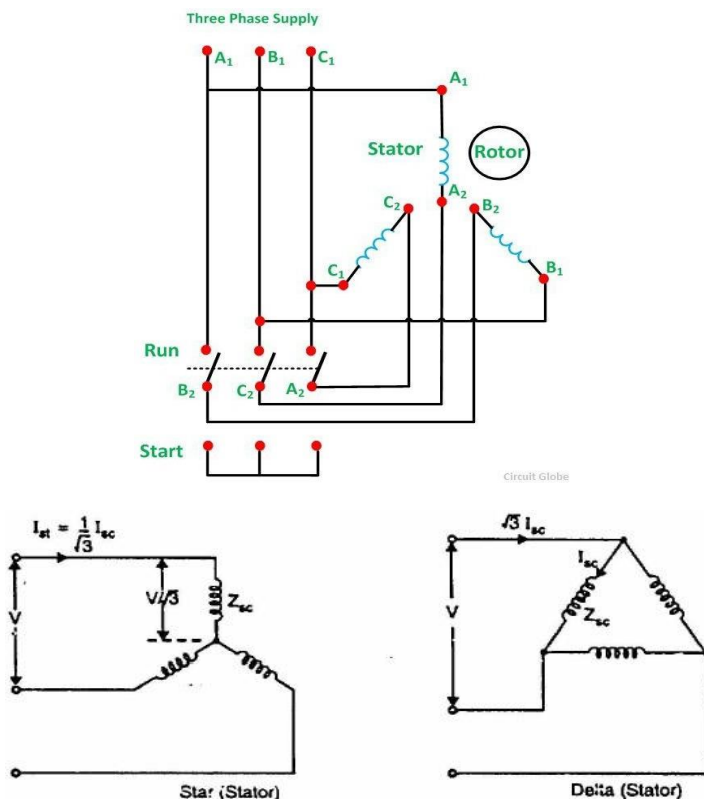
$$\therefore \frac{T_{st}}{T_f} = K^2 \left( \frac{I_{sc}}{I_f} \right)^2 \times s_f$$



The current taken from the supply or by autotransformer is  $I_1 = KI^2 = K^2 I_{sc}$ . Note that motor current is  $K$  times, the supply line current is  $K^2$  times and the starting torque is  $K^2$  times the value it would have been on direct-on-line starting.

#### (iv) Star-delta starting

The stator winding of the motor is designed for delta operation and is connected in star during the starting period. When the machine is up to speed, the connections are changed to delta. The circuit arrangement for star-delta starting is shown in Fig.



The six leads of the stator windings are connected to the changeover switch as shown. At the instant of starting, the changeover switch is thrown to “Start” position which connects the stator windings in star. Therefore, each stator phase gets  $V/\sqrt{3}$  volts where  $V$  is the line voltage. This reduces the starting current. When the motor picks up speed, the changeover switch is thrown to “Run” position which connects the stator windings in delta. Now each stator phase gets full line voltage  $V$ .

**Relation between starting and F.L. torques.** In direct delta starting,

Starting current/phase,  $I_{sc} = V/Z_{sc}$  where  $V$  = line voltage

Starting line current =  $\sqrt{3} I_{sc}$

In star starting, we have,

Starting current/phase,  $I_{st} = \frac{V/\sqrt{3}}{Z_{sc}} = \frac{1}{\sqrt{3}} I_{sc}$

Now 
$$\frac{T_{st}}{T_f} = \left(\frac{I_{st}}{I_f}\right)^2 \times S_f = \left(\frac{I_{sc}}{\sqrt{3} \times I_f}\right)^2 \times S_f$$

or 
$$\frac{T_{st}}{T_f} = \frac{1}{3} \left(\frac{I_{sc}}{I_f}\right)^2 \times S_f$$

where  $I_{sc}$  = starting phase current (delta)  
 $I_f$  = F.L. phase current (delta)

Note that in star-delta starting, the starting line current is reduced to one-third as compared to starting with the winding delta connected. Further, starting torque is reduced to one-third of that obtainable by direct delta starting. This method is cheap but limited to applications where high starting torque is not necessary e.g., machine tools, pumps etc.

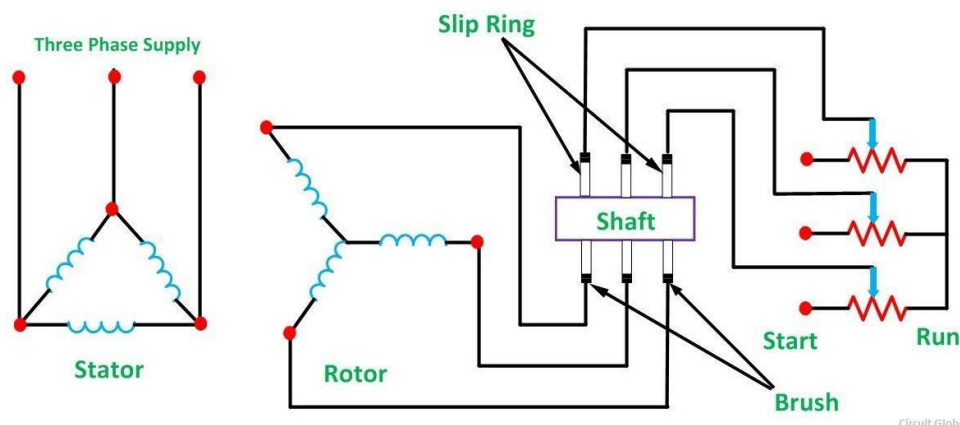
The disadvantages of this method are:

- (a) With star-connection during starting, stator phase voltage is  $1/\sqrt{3}$  times the line voltage. Consequently, starting torque is  $1/3$  or  $1/3$  times the value it would have with  $\Delta$ -connection. This is rather a large reduction in starting torque.
- (b) The reduction in voltage is fixed.

This method of starting is used for medium-size machines (upto about 25 H.P.).

### Starting of Slip-Ring Motors

Slip-ring motors are invariably started by rotor resistance starting. In this method, a variable star-connected rheostat is connected in the rotor circuit through slip rings and full voltage is applied to the stator winding as shown in fig



At starting full starting resistance is connected and thus the supply current to the stator is reduced. The rotor begins to rotate, and the rotor resistances are gradually cut out as the speed of the motor increases. When the motor is running at its rated full load speed, the starting resistances are cut out completely, and the slip rings are short-circuited.

### Slip-Ring Motors Versus Squirrel Cage Motors

The slip-ring induction motors have the following advantages over the squirrel cage motors:

- (i) High starting torque with low starting current.
- (ii) Smooth acceleration under heavy loads.
- (iii) No abnormal heating during starting.
- (iv) Good running characteristics after external rotor resistances are cut out.
- (v) Adjustable speed.

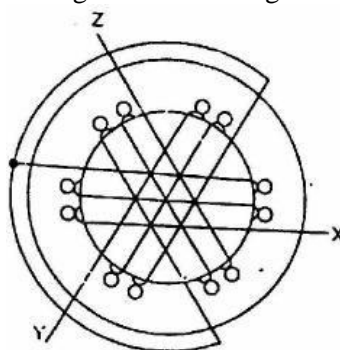
The disadvantages of slip-ring motors are:

- (i) The initial and maintenance costs are greater than those of squirrel cage motors.
- (ii) The speed regulation is poor when run with resistance in the rotor circuit

**Rotating Magnetic Field Due to 3-Phase Currents**

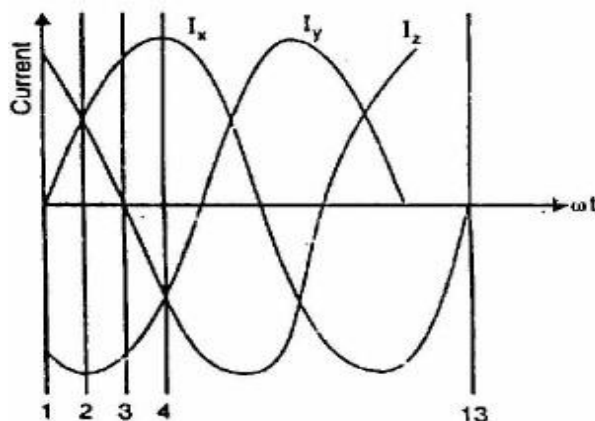
When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do not remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating field. It can be shown that magnitude of this rotating field is constant and is equal to  $1.5 \phi_m$  where  $\phi_m$  is the maximum flux due to any phase.

Consider a 2-pole, 3-phase winding as shown in Fig.



(i)

The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as  $I_x$ ,  $I_y$  and  $I_z$



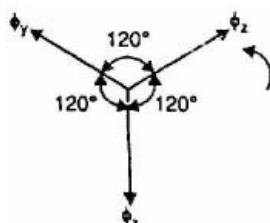
(ii)

The fluxes produced by these currents are given by:

$$\phi_x = \phi_m \sin \omega t$$

$$\phi_y = \phi_m \sin (\omega t - 120^\circ)$$

$$\phi_z = \phi_m \sin (\omega t - 240^\circ)$$



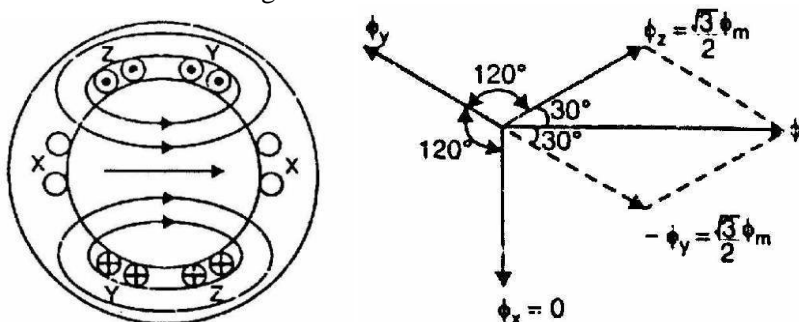
Here  $\phi_m$  is the maximum flux due to any phase. Above Fig. shows the phasor diagram of the three fluxes.



**Proof of this 3-phase supply produces a rotating field of constant magnitude equal to  $1.5 \phi_m$ .**

1. At instant 1 [See Fig. (ii) and below Fig.]

The current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward in the bottom conductors. This establishes a resultant flux towards right.



The magnitude of the resultant flux is constant and is equal to  $1.5 \phi_m$

At instant 1,  $\omega t = 0^\circ$ . Therefore, the three fluxes are given by;

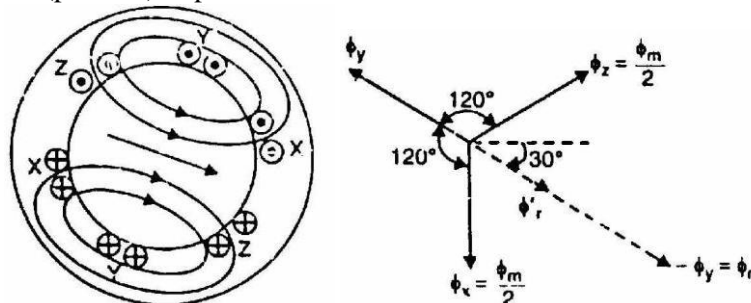
$$\phi_x = 0; \quad \phi_y = \phi_m \sin(-120^\circ) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-240^\circ) = \frac{\sqrt{3}}{2}\phi_m$$

The phasor sum of  $-\phi_y$  and  $\phi_z$  is the resultant flux  $\phi_r$  [See Fig. (8.7)]. It is clear that:

$$\text{Resultant flux, } \phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos \frac{60^\circ}{2} = 2 \times \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2} = 1.5 \phi_m$$

2. At instant 2, [See Fig. (ii) and below Fig.] the current is maximum (negative) in  $\phi_y$  phase Y and 0.5 maximum (positive) in phases X and Z.



The magnitude of resultant flux is  $1.5 \phi_m$

At instant 2,  $\omega t = 30^\circ$ . Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 30^\circ = \frac{\phi_m}{2}$$

$$\phi_y = \phi_m \sin(-90^\circ) = -\phi_m$$

$$\phi_z = \phi_m \sin(-210^\circ) = \frac{\phi_m}{2}$$

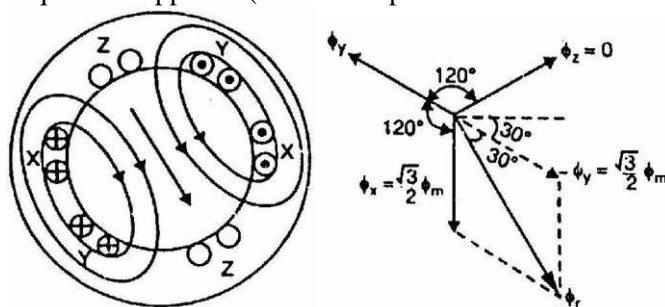
The phasor sum of  $\phi_x$ ,  $-\phi_y$  and  $\phi_z$  is the resultant flux  $\phi_r$

$$\text{Phasor sum of } \phi_x \text{ and } \phi_z, \phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}$$

$$\text{Phasor sum of } \phi'_r \text{ and } -\phi_y, \phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$$

Note that resultant flux is displaced  $30^\circ$  clockwise from position 1.

3. At instant 3, [See Fig. (ii) and below Fig.] current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y are  $0.866 \times$  max. value).



The magnitude of resultant flux is  $1.5 \phi_m$

At instant 3,  $\omega t = 60^\circ$ . Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 60^\circ = \frac{\sqrt{3}}{2} \phi_m;$$

$$\phi_y = \phi_m \sin(-60^\circ) = -\frac{\sqrt{3}}{2} \phi_m;$$

$$\phi_z = \phi_m \sin(-180^\circ) = 0$$

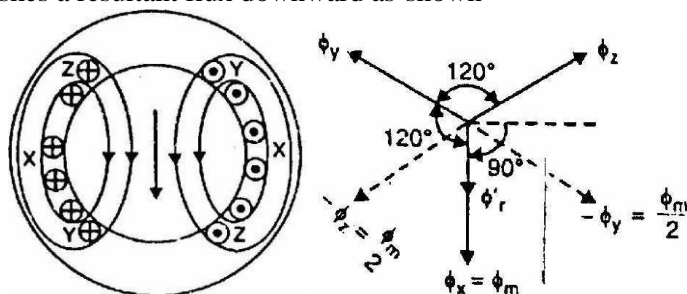
The resultant flux  $\phi_r$  is the phasor sum of  $\phi_x$  and  $-\phi_y$  ( $\because \phi_z = 0$ ).

$$\phi_r = 2 \times \frac{\sqrt{3}}{2} \phi_m \cos \frac{60^\circ}{2} = 1.5 \phi_m$$

Note that resultant flux is displaced  $60^\circ$  clockwise from position 1.

Fig.(8.9)

4. At instant 4, [See Fig. (ii) and below Fig.] the current in phase X is maximum (positive) and the currents in phases Y and Z are equal and negative (currents in phases Y and Z are  $0.5 \times$  max. value). This establishes a resultant flux downward as shown



At instant 4,  $\omega t = 90^\circ$ . Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 90^\circ = \phi_m$$

$$\phi_y = \phi_m \sin(-30^\circ) = -\frac{\phi_m}{2}$$

$$\phi_z = \phi_m \sin(-150^\circ) = -\frac{\phi_m}{2}$$

The phasor sum of  $\phi_x$ ,  $-\phi_y$  and  $-\phi_z$  is the resultant flux  $\phi_r$

$$\text{Phasor sum of } -\phi_z \text{ and } -\phi_y, \phi'_r = 2 \times \frac{\phi_m}{2} \cos \frac{120^\circ}{2} = \frac{\phi_m}{2}$$

$$\text{Phasor sum of } \phi'_r \text{ and } \phi_x, \phi_r = \frac{\phi_m}{2} + \phi_m = 1.5 \phi_m$$

Note that the resultant flux is downward i.e., it is displaced  $90^\circ$  clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value ( $= 1.5 \phi_m$ , where  $\phi_m$  is the maximum flux due to any phase).

### Circle Diagram of Induction Motor

The circle diagram of an induction motor is very useful to study its performance under all operating conditions. The “CIRCLE DIAGRAM” means that it is figure or curve which is drawn has a circular shape. As we know, the diagrammatic representation is easier to understand and remember compared to theoretical and mathematical descriptions.

#### Importance of Circle Diagram

The diagram provides information which is not provided by an ordinary phasor diagram. A phasor diagram gives relation between current and voltage only at a single circuit condition. If the condition changes, we need to draw the phasor diagram again. But a circle diagram may be referred to as a phasor diagram drawn in one plane for more than one circuit conditions. On the context of induction motor, which is our main interest, we can get information about its power output, power factor, torque, slip, speed, copper loss, efficiency etc. in a graphical or in a diagrammatic representation.

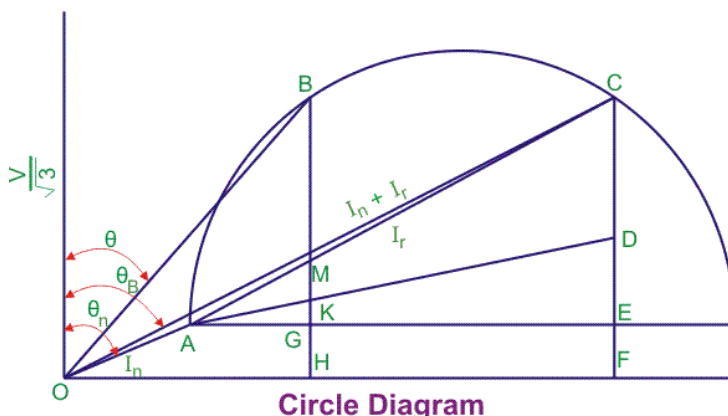
#### Test Performed to Compute Data Required for Drawing Circle Diagram

We have to perform no load and blocked rotor test in an induction motor. In no load test, the induction motor is run at no load and by two watt meter method, its total power consumed is calculated which is composed of no load losses only. Slip is assumed to be zero. From here no load current and the angle between voltage and current required for drawing circle diagram is calculated. The angle will be large as in the no load condition induction motor has high inductive reactance.

#### Procedure to Draw the Circle Diagram

We have to assume a suitable before drawing it. This assumption is done according to our convenience.

1. The no load current and the no load angle calculated from no load test is plotted. This is shown by the line OA, where  $\Theta_0$  is the no load power factor angle.
2. The short circuit current and the angle obtained from block rotor test is plotted. This is shown by the line OC and the angle is shown by  $\Theta_B$ .
3. The right bisector of the line AC is drawn which bisects the line and it is extended to cut in the line AE which gives us the centre.
4. The stator current is calculated from the equivalent circuit of the induction motor which we get from the two tests. That current is plotted in the circle diagram according to the scale with touching origin and a point in the circle diagram which is shown by B.
5. The line AC is called the power line. By using the scale for power conversion that we have taken in the circle diagram, we can get the output power if we move vertically above the line AC to the periphery of the circle. The output power is given by the line MB.
6. The total copper loss is given by the line GM.
7. For drawing the torque line, the total copper loss should be separated to both the rotor copper loss and stator copper loss. The line DE gives the stator copper loss and the line CD gives the rotor copper loss. In this way, the point E is selected.
8. The line AD is known as torque line which gives the torque developed by induction motor.



**Maximum Quantities from Circle Diagram**

**Maximum Output Power**

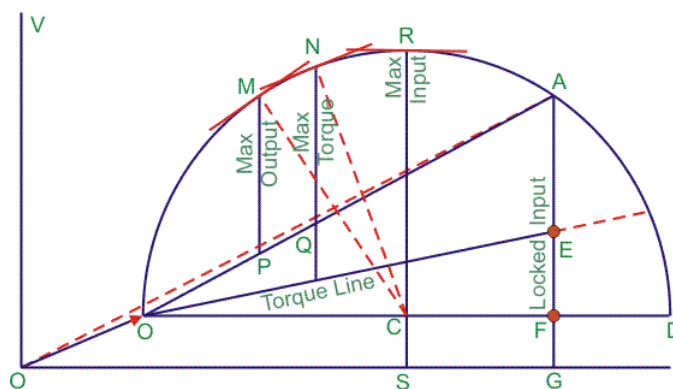
When the tangent to the circle is parallel to the line then output power will be maximum. That point M is obtained by drawing a perpendicular line from the center to the output line and extending it to cut at M.

**Maximum Torque**

When the tangent to the circle is parallel to the torque line, it gives maximum torque. This is obtained by drawing a line from the center in perpendicular to the torque line AD and extending it to cut at the circle. That point is marked as N.

**Maximum Input Power**

It occurs when tangent to the circle is perpendicular to the horizontal line. The point is the highest point in the circle diagram and drawn to the center and extends up to S. That point is marked as R.



**Conclusion of Circle Diagram**

This method is based on some approximations that we have used in order to draw the circle diagram and also, there is some rounding off of the values as well. So there is some error in this method but it can give good approximate results. Also, this method is very much time consuming so it is drawn at times where the drawing of circle diagram is absolutely necessary. Otherwise, we can go for mathematical formulas or equivalent circuit model in order to find out various parameters.

## Single Phase Induction Motor

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The single-phase induction machine is the most frequently used motor for domestic as well as industrial low power application. These motors are also known as *fractional KW motors* because most of these motors are constructed in fractional kilowatt capacity. Single phase induction motors require just one power phase for their operation. The single phase motors are widely seen in many applications as it is simple in construction, cheap in cost, reliable and easy to repair and maintain. Due to these advantages, the single phase motor is used in vacuum cleaner, fans, washing machine, centrifugal pump, blowers, etc.

The main disadvantage of single phase induction motor are low output, low overload capacity, low power factor and also low efficiency compared to three phase induction motor. Also a single phase induction motor is not self starting.

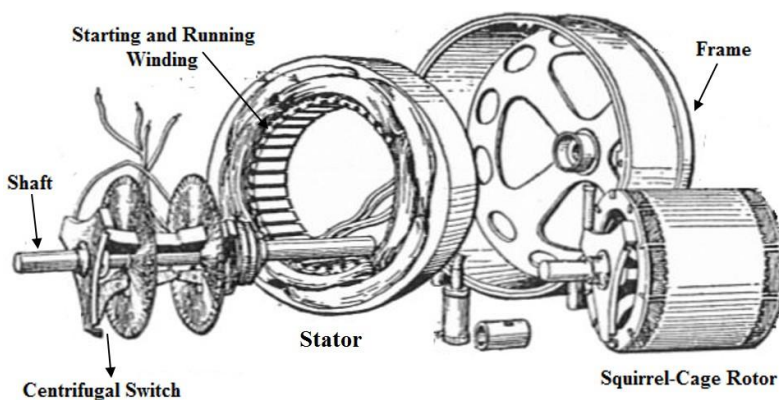


Figure 5.1: Dismantled Single Phase Induction Motor

The construction of a single phase induction motor is similar to the construction of three phase induction motor having squirrel cage rotor, except that the stator is wound for single phase supply. An additional winding known as *starting winding* or *auxiliary winding* is also provided with stator to make the motor self starting. So it can be temporarily converted into a two-phase motor while starting. These two windings are spaced  $90^\circ$  electrical degrees apart and are connected in parallel across a single phase supply. The Phase difference of  $90^\circ$  degree is obtained by connecting a capacitor in series with the starting winding.

### 5.1 Principle of Operation

When the stator winding of single phase induction motor is given a single phase AC supply, the current starts flowing through the stator which is alternating in nature. This alternating current produces an alternating flux which is pulsating but not rotating like three phase induction motor. This alternating flux also links with the rotor conductors and hence the rotor conductors cut the magnetic flux. According to the Faraday's law of electromagnetic induction, emf gets induced in the rotor causes induced current in the rotor bars. This induced current also produce rotor flux which react against the stator field. So the result is a pulsating torque. So a single phase induction motor is not self starting.

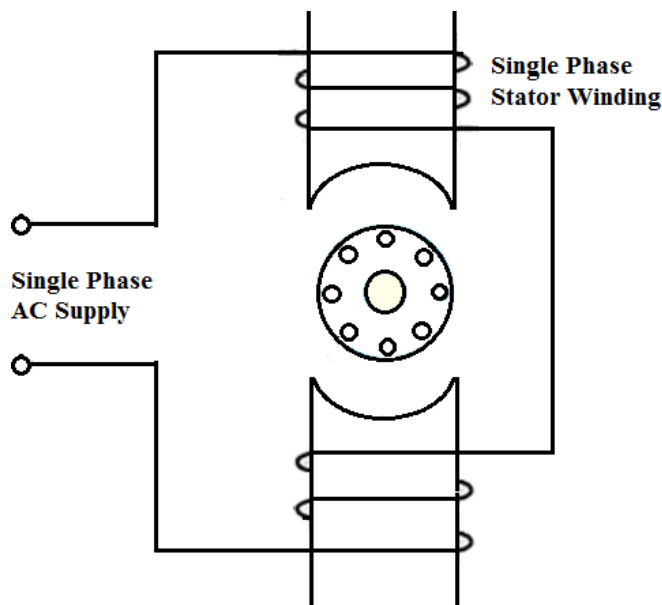


Figure 5.2: Single Phase Induction Motor

However, if the rotor is given a initial start by external force in either direction, it develops an initial torque then motor accelerates to its final speed and keeps running with its rated speed. Thus single phase induction motor is not self starting and need special design to make it self starting.

## 5.2 Why Single phase induction motor is not self starting

According to double field revolving theory, any alternating field produced in a single phase induction motor can be resolved into two components. These two component having half its maximum magnitude and rotates in opposite direction to each other.

So the flux,  $\phi$  can be resolved into two components  $+\frac{\phi}{2}$  and  $-\frac{\phi}{2}$ . So one component rotating in clockwise direction and other component in anti-clockwise direction. So the resultant of these two flux is zero.

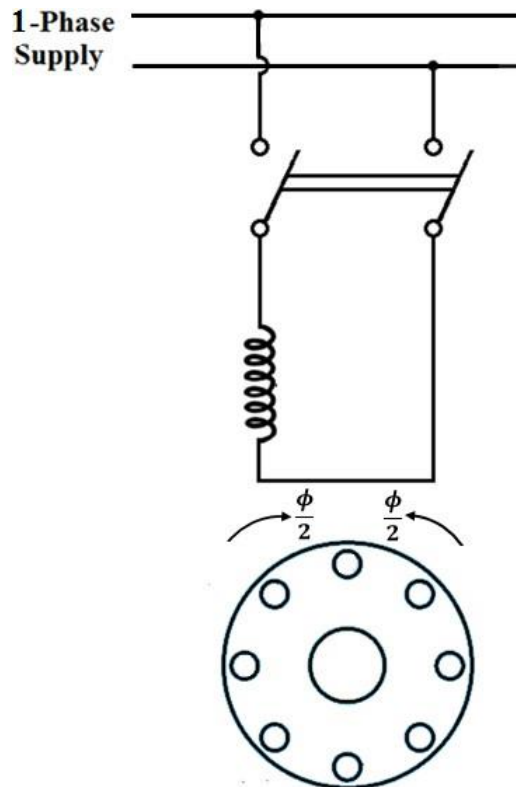


Figure 5.3: Double Field Revolving Theory

The two component fluxes while revolving, cuts the rotor conductors and induces emf and produces its own torque. The two torques are known as forward torque  $T_f$  and backward torque  $T_b$ . The two torques are equal in magnitude and rotating in opposite direction, the resultant torque is zero. So the single phase induction motor is not self starting.

However if the rotor is given an external pull in any direction, the torque towards that direction increases and the torque towards the opposite direction decreases. So there is a net torque in one direction which accelerates the motor in that direction.

### 5.3 Methods for Making Single Phase Induction as Self Starting Motor

Single phase induction motor is not self starting. In order to make the motor self-starting, it is temporarily converted into a two-phase motor during starting period. When the stator winding is wound for two phase, stator flux becomes rotating type, rather than alternating type. This would result in an unidirectional torque. Then the induction motor will become self starting.

The stator of a single-phase motor consists of an extra winding, known as starting (or auxiliary) winding, in addition to the main or running winding. These two windings are connected in parallel across the single-phase supply. These two windings are having equal number of turns and placed at a space angle of  $90^\circ$  (electrical).

Depending upon the methods for making single phase induction motor as Self Starting Motor, it is classified as,

#### 5.3.1 Split Phase Motor

In this motor, main winding is wound many turns to have higher reactance and lower resistance. A high resistance  $R$  is connected in series with the starting winding so the starting winding has high resistance, but low reactance. The starting winding is placed at a space angle of  $90^\circ$  from the main winding.

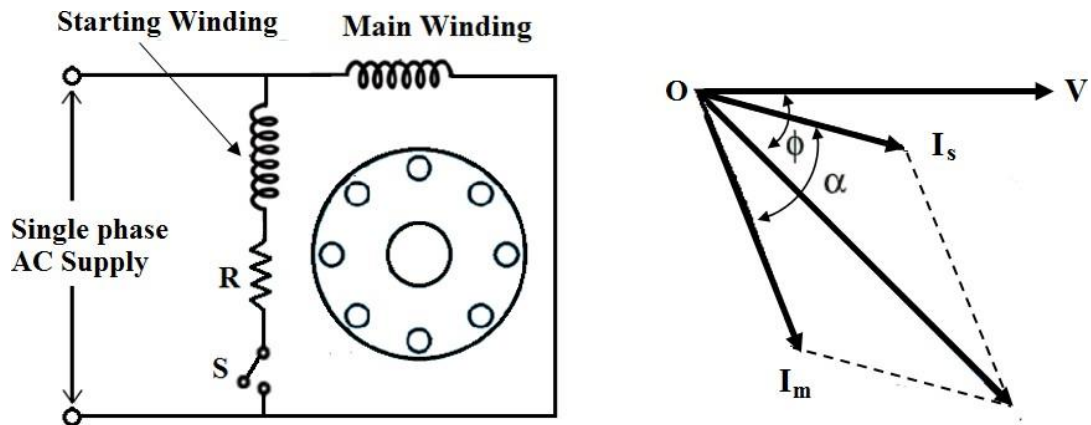


Figure 5.4: Split Phase Motor

The phasor diagram of the currents in two windings and the input voltage is shown in Figure 5.4. Due to high resistance and low reactance, the current  $I_s$  taken by the starting winding lags behind the applied voltage  $V$  by a small angle.

But due to low resistance high reactance, the current  $I_m$  taken by the main winding lags behind  $V$  by a very large angle. So there is a definite Phase angle  $\alpha$  between  $I_m$  and  $I_s$ . The phase angle between the two currents should be at least  $30^\circ$ . This results in a small amount of starting torque. The angle is made as large as possible because the starting torque of a split-phase motor is proportional to  $\alpha$ .



## Single Phase Induction Motor

A centrifugal switch  $S$  is provided with the starting winding. When the motor reaches 70% to 80% of its full-load speed, the centrifugal switch automatically disconnects the starting winding from the supply.

These motors have low starting torque. So it is used in fans, blowers, centrifugal pumps, etc.

### 5.3.2 Capacitor Start Motor

In a capacitor start motor, the capacitor of suitable value is connected in series with the auxiliary winding. Capacitor is used to improve the starting performance of the single phase induction motors. An electrolytic type capacitor is commonly used and it is usually placed outside. A centrifugal switch is connected in series with the starting winding and capacitor. The main function of centrifugal switch is it disconnects the starting winding and capacitor from the supply when the motor reaches about 75% of full speed.

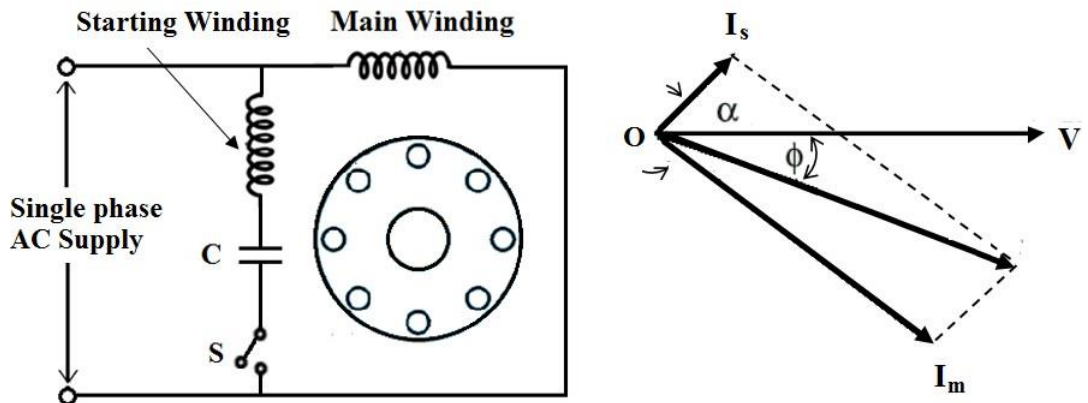


Figure 5.5: Capacitor Start Motor

The phasor diagram of the currents in the two windings and the input voltage is shown in Figure 5.5. Due to the capacitor connected in series with the starting winding, the current  $I_s$  taken by the starting winding leads the applied voltage  $V$  by certain angle. The current  $I_m$  drawn by the main winding lags behind the supply voltage  $V$  by a large angle. The two currents are out of phase with each other by about  $80^\circ$  as compared to nearly  $30^\circ$  for a split-phase motor. This results in a very high starting torque, because the starting torque of a split-phase motor is proportional to  $\alpha$ .

These motors have very high starting torque. So it is used in compressor, air-conditioner, grinders, conveyors etc.

### 5.3.3 Capacitor start capacitor run motor

In Capacitor start capacitor run motor, the starting winding and capacitor is permanently connected to the circuit at all times. Here the starting winding and capacitor

are to be so designed that the motor works at anyone desired load .

There are two types of Capacitor start capacitor run motor. In one kind of motor, the motor starts and run with one capacitor in the circuit and is called single-value capacitor motor. In second type, the motor starts with high value of capacitance but run with a low value of capacitance are known as two-value capacitor-run motors.

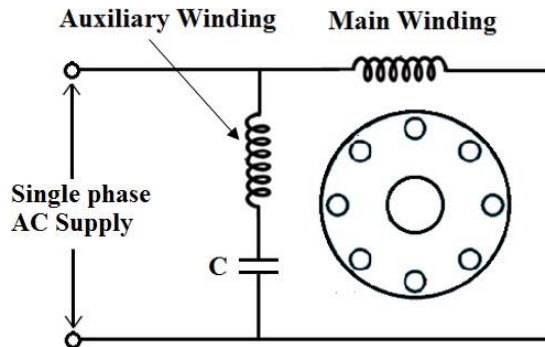


Figure 5.6: Capacitor start capacitor run motor

The advantages of connecting capacitor permanently to the circuit are

- (i) It increases the over-load capacity of the motor
- (ii) It improves the power factor
- (iii) Efficiency increases
- (iv) Quite operation
- (v) No centrifugal switch is needed
- (vi) Higher pull out torque.
- (vii) Improvement of starting and running performance

### 5.3.4 Shaded Pole motor

The stator of the shaded pole single phase induction motor has salient or projected poles. These poles are shaded by copper band or ring which is inductive in nature. The poles are divided into two unequal halves. The smaller portion carries the copper band and is called as shaded portion of the pole.

When a single phase supply is given to the stator of shaded pole induction motor an alternating flux is produced. This change of flux induces emf in the shaded coil. Since this shaded portion is short circuited, the current is produced in it in such a direction to oppose the main flux. The flux in shaded pole lags behind the flux in the unshaded pole. The phase difference between these two fluxes produces resultant rotating flux.

## Single Phase Induction Motor

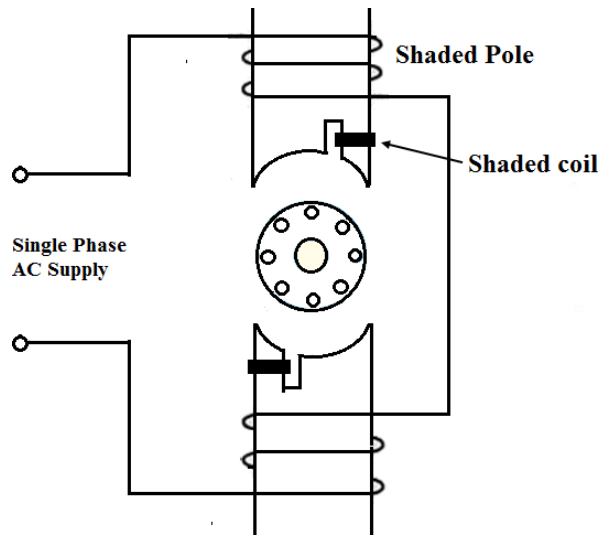


Figure 5.7: Shaded Pole Motor

Due to their low starting torques and reasonable cost these motors are mostly employed in small instruments, hair dryers, toys, record players, small fans, electric clocks etc.

### Note

What happens if the centrifugal switch fails to operate?

If it fails to open, the motor will over heat the main winding and if it fails to close, it will overheat the motor without any damage to main winding.

Table 5.1: Comparison of 3-Phase and 1-Phase Induction Motor

	3-Phase Induction Motor	1-Phase Induction Motor
1	Three phase supply is used	Single phase supply is used
2	It is self-starting	It is not self-starting
3	Starting torque is high	Starting torque is low
4	Complex in construction	Simple in construction
5	Efficiency is high	Efficiency is low
6	Power factor is high	Power factor is low
7	Difficult to repair and maintain	Easy to repair and maintain
8	Centrifugal switch is absent	Centrifugal switch is used
9	Expensive	Cheap
10	Auxiliary winding is not required	Auxiliary winding required for starting

**EXERCISE**

1. Explain the principle of operation of a  $1-\phi$  induction motor ?
2. Explain split phase motor ?
3. Why single phase induction motor is not self starting ?
4. Explain the function of a centrifugal switch in an induction motor ?
5. What are fractional kilowatt motors ?
6. Draw and explain the phasor diagram of a split phase motor ?
7. Compare single phase and three phase induction motor ?
8. Explain different methods to make a single phase induction motor self starting ?
9. Explain capacitor start capacitor run motor ?
10. With the help of phasor diagram explain capacitor start motor ?
11. What happens if the centrifugal switch failed to operate.
12. Explain shaded pole motor ?
13. What are the advantages of adding capacitor permanently in a single phase induction motor ?

## Synchronous Machines

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A synchronous machine is an AC rotating machine whose speed is proportional to the frequency of the current in its armature. In a synchronous machine, the magnetic field created by the armature currents rotates at the same speed as that of the rotor, which is rotating at the synchronous speed. Synchronous machines are commonly used as AC generators or alternators. They are used to generate energy in power plants to supply the electric power to industrial, commercial, agricultural, and domestic applications. For the purpose of power generation, these machines are built for higher rating. The prime movers are usually steam turbines in the case of high-speed machine and for low speed machines, usually hydraulic turbines are used.

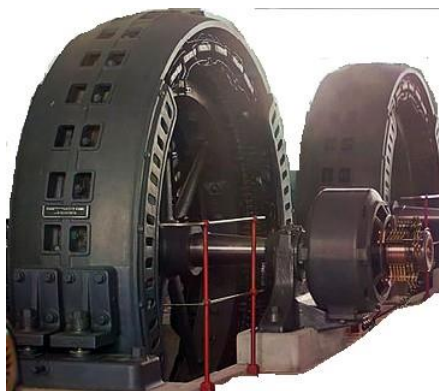


Figure 6.1: Alternator

In alternator, the armature winding is connected to the AC supply and the field winding is connected to DC supply for excitation. The stationary armature acts as

the stator and it is housed with three-phase armature winding. The field winding is on the rotor and is excited.

Synchronous machines can also be used as a motor which operates at synchronous speed. So synchronous motor is a constant speed motor. The synchronous motor has a variable-power-factor characteristics, and hence is suitable for power-factor correction. It's stator winding is connected to AC supply and it's field winding is connected to DC supply.

According to the way in which armature and field windings are arranged, the synchronous machines are classified as rotating armature type or rotating field type machine. In rotating armature type, the field winding is on the stator and the armature winding is on the rotor. This kind of construction is mainly adopted for small sized machines. In case of rotating field type generators, the armature windings are on the stator and field windings are on the rotor.

### 6.1 Advantages of Stationary Armature and Rotating Field System

Medium and large-sized machine are normally constructed with stationary armature and rotating field system. Advantages of having stationary armature are :

1. It is easier to insulate the stationary armature winding for very high voltages.
2. The output current can be directly taken from the fixed armature terminals, without passing through the slip rings and brushes.
3. The armature windings can be more easily braced to prevent any deformation, which could be developed by the mechanical stresses due to short-circuit current and the high centrifugal forces brought into play.
4. Armature winding can be cooled more easily because of the presence of very large stator with many air passages or cooling ducts for cooling purpose
5. Due to robust construction, slip-rings are connected to the low-voltage, DC field circuit which can, therefore, be easily insulated.

### 6.2 Construction of Alternator

Synchronous machines essentially consist of two parts namely stator and rotor

#### 6.2.1 Stator

The armature is built up of laminations of special magnetic iron or steel alloy. The core is laminated to minimise eddy current loss. Laminations are insulated from each other. The slots are placed on the periphery of the armature to house armature windings. For the purpose of cooling, spaces are provided for the cooling air to pass through. The stator is wound with a 3 phase winding. The stator of a typical alternator is shown in Figure 6.2.



Figure 6.2: Stator of Alternator

### 6.2.2 Rotor

Two types of rotors are used in alternators (i) salient-pole type and (ii) smooth-cylindrical type.

#### Salient-Pole Rotor

---

Salient pole rotor consists of salient or projecting poles that means the poles are projecting out from the surface of the rotor. It is generally used in low-and medium-speed alternators. Hydraulic turbines and IC engines are used as prime-movers. Such generators are characterised by their large diameters and short axial lengths. The poles and pole-shoes are laminated to minimize eddy current loss. The pole shoes covers only about  $2/3^{rd}$  of pole pitch. In large machines, field windings consist of rectangular copper strip wound on edge. The rotor core is bolted or dovetailed onto a heavy magnetic wheel of cast-iron or cast steel. So the rotor resembles a flywheel with electromagnets. This flywheel type rotor construction will provide sufficient mechanical strength to the rotor for withstanding centrifugal and driving forces.

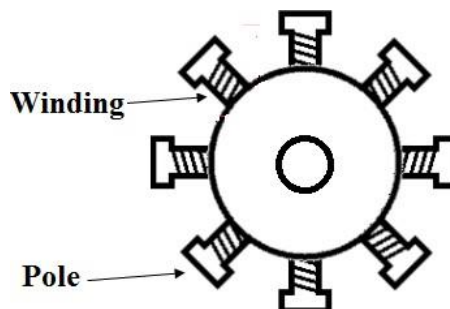


Figure 6.3: Salient Pole

Salient pole machine is cheaper than cylindrical rotor machine when the speed is below 1000 r.p.m. Salient pole rotor cannot be used in high speed machines due to high peripheral speed and low mechanical strength. Damper windings consist of copper bars are used to damp rotor oscillation during transient condition.

The salient pole rotor has the following salient features.

- / Used in low-and medium-speed alternators.
- / They have a larger diameter and shorter axial length.
- / The pole shoes covers only about  $2/3^{rd}$  of the pole pitch.
- / Poles are laminated to reduce eddy current loss.
- / Damper windings are used to damp rotor oscillation during transient condition.
- / Mechanical stability is less compared to smooth cylindrical rotor.
- / Rotor construction results in non uniform air gap

### Smooth Cylindrical Rotor

---

Smooth cylindrical rotor is also known as non-salient pole rotor because the poles are not projecting out from the surface of the rotor. It is generally used in very high speed, steam turbine-driven alternators known as *turbo-alternator* . In order to reduce excessive peripheral velocity, such rotors are characterised by small diameters and very long axial length. To reduce mechanical stresses, rotor consists of a smooth solid forged steel cylinder, having a number of slots milled out at intervals along the outer periphery for housing field coils. The cylindrical rotor construction of the rotor offers better dynamic balance and quieter-operation. It also reduces windage losses.

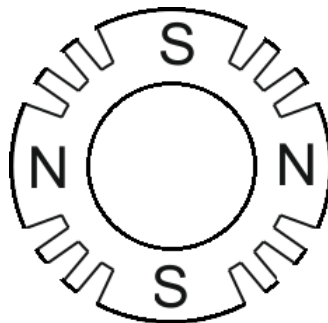


Figure 6.4: Smooth Cylindrical Rotor



Such rotors are designed mostly for 2-pole or 4-pole turbo-generators running at 3600 r.p.m. or 1800 r.p.m. Two or four regions corresponding to the central polar

## Synchronous Machines

areas are left unslotted. The unslotted portion of the cylinder itself acts as the poles, as shown in Figure 6.4

The salient pole rotor has the following salient features.

- / Used in high-speed turbo-alternators.
- / They have a smaller diameter and longer axial length.
- / Noiseless operation
- / Better in dynamic balancing.
- / Damper windings are not required.
- / Windage losses are less
- / Two or four regions corresponding to the central polar areas are left unslotted and the unslotted portion of the cylinder acts as the poles.
- / Rotor construction results in uniform air gap

**Table 6.1: Comparison of Salient-Pole and Smooth Cylindrical type of Rotor**

	<b>Salient Pole Type</b>	<b>Smooth Cylindrical Type</b>
1	Salient or Projecting Pole	Non-salient poles where unslotted portion of the cylinder acts as the poles
2	They have a larger diameter and short axial length.	They have a smaller diameter and longer axial length.
3	Used in low and medium speed machines	Used in high-speed turbo-alternators.
4	Poor Mechanical stability	Better mechanical stability
5	Hydraulic turbines and IC engines are used as prime movers	Steam turbines are commonly used as prime-movers
6	Non-uniform airgap	Uniform airgap
7	Damper windings are provided	Damper windings are not necessary
8	For same size, rating is less	For same size, rating is high

9	More windage loss	Less windage loss
10	Less expensive	Expensive

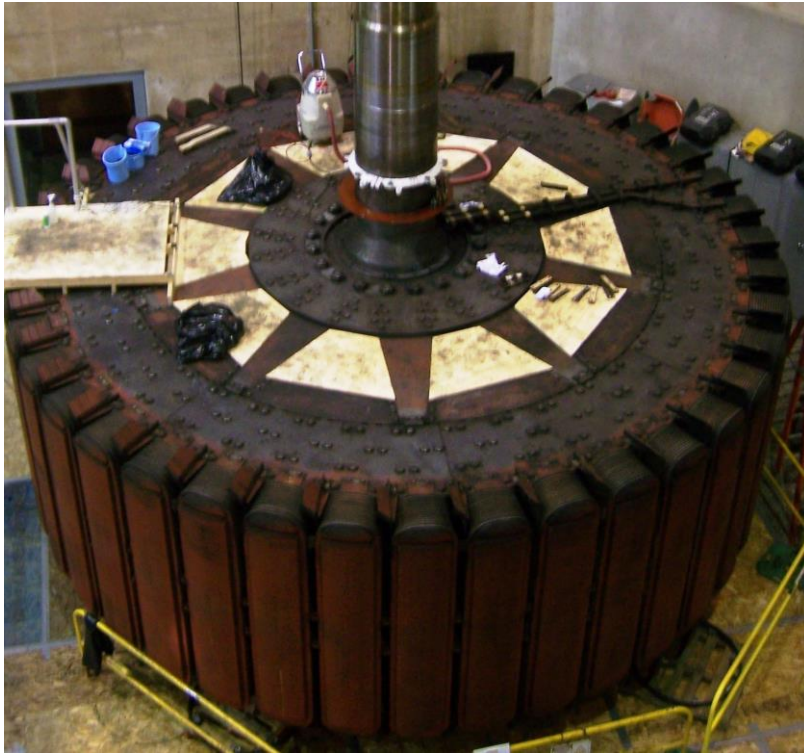


Figure 6.5: Salient Pole Rotor

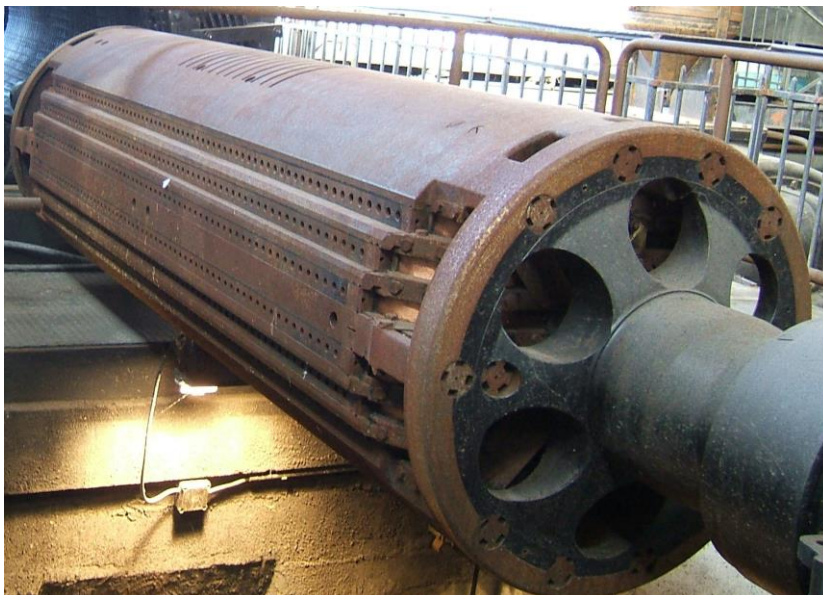


Figure 6.6: Smooth Cylindrical Rotor

## Synchronous Machines

### 6.3 Emf Equation

Let,

- $\Phi$  = Useful flux/pole in weber
- $f$  = Frequency of induced emf in Hertz
- $P$  = Number of pole
- $Z$  = Number of conductors in series per phase
- $T$  = Number of turns/phase =  $\frac{Z}{2}$
- $N$  = Speed in r.p.m.
- $K_p$  or  $K_c$  = Coil span factor or pitch factor
- $K_d$  = Distribution factor

Total flux cut by each stator conductor is  $\Phi P$  webers. The time for one revolution of the rotor is given by  $\frac{60}{N}$

$$\begin{aligned}\text{Average e.m.f. induced per conductor} &= \frac{d\Phi}{dt} \\ &= \frac{\Phi P}{60/N} \\ &= \frac{\Phi N P}{60} \quad \text{volt}\end{aligned}$$

The rotational speed of the alternator is given by,

$$\begin{aligned}N &= \frac{120f}{P} \\ \therefore \text{Average emf induced per conductor} &= \frac{\Phi P}{60} \times \frac{120f}{P} \\ &= 2f\Phi \quad \text{volt}\end{aligned}$$

If there are  $Z$  conductors in series/phase,

$$\begin{aligned}\therefore \text{Average induced emf per phase} &= 2f\Phi Z \\ &= 4f\Phi T \quad \text{volt} \\ \text{r.m.s value of emf per phase} &= 1.11 \times 4f\Phi T \\ &= 4.44f\Phi T \quad \text{Volt}\end{aligned}$$

$$\boxed{\therefore E_{rms}/\text{phase} = 4.44f\Phi T} \quad \text{Volt} \quad (6.1)$$

For short pitch and distributed winding

$$\boxed{E_{rms}/\text{phase} = 4.44 K_c K_d f \Phi T} \quad \text{Volt} \quad (6.2)$$

**Note**

The voltage induced in the armature coils depends on two winding factors:

**Pitch Factor ( $K_p$ ) :** In short-pitched coils, there is a reduction in voltage around the coils. The voltages induced in the two sides of the short-pitched coil are not in phase. Due to this, the resultant vector sum of these two voltages is less than the arithmetical sum. This difference can be illustrated with the help of pitch factor.

The pitch factor ( $K_p$ ) is defined as,

$$K_p = \frac{\text{Vector sum of the induced e.m.f.s. per coil}}{\text{Arithmetic sum of the induced e.m.f.s. per coil}} \quad (6.3)$$

if the coil is short pitched by an electrical angle  $\alpha$ ,

$$K_p = \cos \alpha \quad (6.4)$$

For a full-pitch winding,  $K_p = 1$ . However, for a short-pitch winding,  $K_p < 1$ .

**Distribution Factor ( $K_d$ ) :** If the armature winding of alternator has only one slot per pole per phase, the winding is said to be a concentrated winding. In concentrated winding, the e.m.f. generated/phase is equal to the arithmetic sum of the individual coil e.m.f.s in that phase. However, if the coils/phase are distributed over several slots in space, the winding is said to be distributed winding. The e.m.f./phase will be the phasor sum of coil e.m.f.s. which are not in phase. The distribution of winding is illustrated in the form of distribution factor.

The distribution factor  $K_d$  is defined as,

$$K_d = \frac{\text{E.m.f. with distributed winding}}{\text{E.m.f. with concentrated winding}} \quad (6.5)$$

Let  $\beta$  of angular displacement between the slots,  $m$  is the number of slots/phase/pole,

$$K_d = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{\beta}{2}} \quad (6.6)$$

Where  $\beta$  is obtained as,

$$\begin{aligned} \beta &= \frac{180^\circ}{\text{No. of slots/pole}} \\ &= \frac{180^\circ}{n} \end{aligned}$$

## Synchronous Machines

### 6.4 Voltage Regulation

When the alternator is loaded, the terminal voltage reduces with increase in load. So the terminal voltage of an alternator changes from no load to full load. Voltage regulation is usually expressed as the change in voltage from a no-load condition to full-load condition, and is expressed as a percentage of the rated terminal voltage. It is expressed in the following formula

$$\text{Voltage regulation} = \frac{E_0 - V}{V} \quad (6.7)$$

$$\text{Percentage Voltage regulation} = \frac{E_0 - V}{V} \times 100 \quad (6.8)$$

where  $E_0$  is the no-load terminal voltage and  $V$  is the full-load terminal voltage of the alternator. Performance of an alternator can be expressed in terms of voltage regulation. Lower the voltage regulation better is the machine.

#### Note

- For leading power factor, no load voltage  $E_0$  is less than full load terminal voltage  $V$ . So the voltage regulation is negative.
- For lagging power factor, no load voltage  $E_0$  is greater than full load terminal voltage  $V$ . So the voltage regulation is positive.

### 6.5 Regulation of alternator by emf method or Synchronous impedance method

The regulation of a 3-phase alternator can be predetermined by conducting the Open Circuit test and the Short Circuit test. The Open circuit characteristics (OCC) and short circuit characteristics (SCC) are plotted from the two tests. The synchronous impedance is found from the OC test.

Let  $E_1$  be the open circuit voltage corresponds to a particular value of field current  $I_f$  and its value is obtained from OCC and  $I_1$  is the short circuit current producing the rated voltage corresponding to the field current  $I_f$  as shown in Figure 6.7.

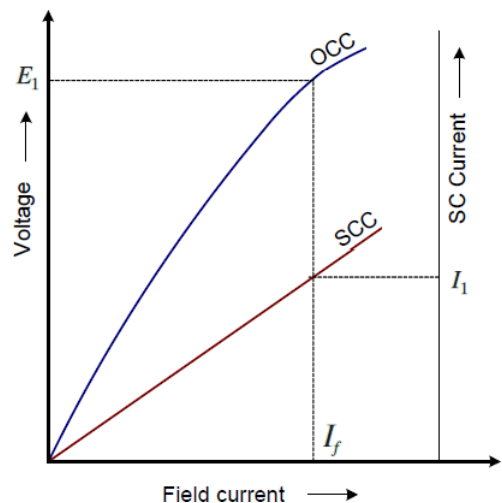


Figure 6.7: •



$$\text{Synchronous impedance, } Z_s = \frac{E_1}{I_1} \quad (6.9)$$

$$\text{Synchronous reactance, } X_s = \frac{Z_s^2 - R^2}{a} \quad (6.10)$$

After determining  $Z_s$  and  $X_s$ , phasor diagram for any load can be drawn as shown in Figure 6.8. Let  $I$  be the load current lagging behind the terminal voltage  $V$  by an angle  $\phi$ .

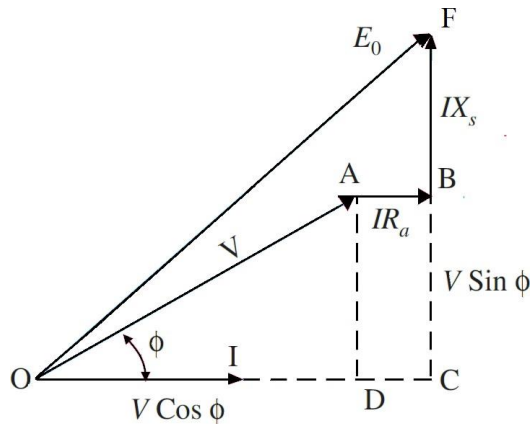


Figure 6.8: •

From the phasor diagram shown in Figure 6.8,

No load terminal voltage,  $E_0 = OF$

Full load terminal voltage,  $V = OA$

Armature resistance drop,  $IR_a = AB$

Synchronous reactance drop,  $IX_s = FB$

From the phasor diagram,

$$\begin{aligned} OF &= \sqrt{OC^2 + FC^2} \\ &= \sqrt{(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_s)^2} \\ \therefore E_0 &= \sqrt{(V \cos \phi + IR_a)^2 + (V \sin \phi + IX_s)^2} \\ \text{and } \% \text{ Regulation} &= \frac{E_0 - V}{V} \times 100 \end{aligned}$$

### Note

Determination of voltage regulation by emf method is not accurate because the value of  $Z_s$  obtained is always more than its value under normal voltage conditions and saturation. Hence, the value of regulation so obtained is always more than the actual value. So it is called *pessimistic method*.

## Synchronous Machines

### 6.6 Synchronous Motor

Synchronous motors are synchronous machines used to convert AC electrical energy to mechanical energy. It is a constant speed motor whose speed is proportional to supply frequency. A synchronous motor is similar to an alternator with a rotating field. Synchronous motors are capable of running at constant speed irrespective of the load acting on them. Unlike induction motors where speed of the motor depends upon the torque acting on them, synchronous motors have got constant speed-torque characteristics.

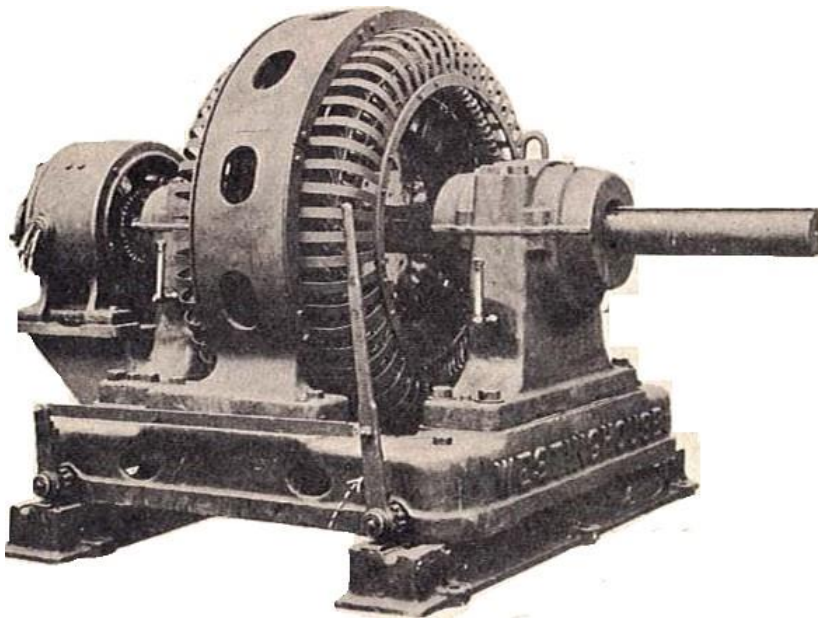


Figure 6.9: Synchronous Motor

Some characteristic features of a synchronous motor are worth noting :

- ☛ It is a constant speed motor and the speed is proportional to supply frequency.
- ☛ It is not inherently self-starting. It has to be run upto synchronous speed by some means, before it can be synchronized to the supply.
- ☛ This motor has the unique characteristics it is capable of being operated under a wide range of power factors, both lagging and leading. This makes it being used in electrical power factor improvement.
- ☛ Synchronous motor is a doubly excited machine i.e the stator is connected to three phase AC supply and the rotor is provided with DC supply.

☞ Synchronous motors have got higher efficiency than induction motor.

6.7 Principle of Operation of Synchronous Motor

When the stator of a three phase synchronous motor is fed by a 3- $\phi$  supply, a magnetic flux of constant magnitude but rotating at synchronous speed is produced similar to that of an induction motor. The rotor carrying DC supply also produces a constant flux. Consider a synchronous motor having two pole stator as shown in Figure 6.10. The two stator poles rotating at synchronous speed, say, in anticlockwise direction.

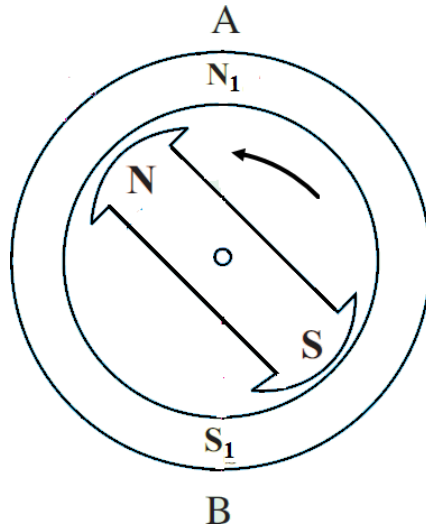


Figure 6.10: •

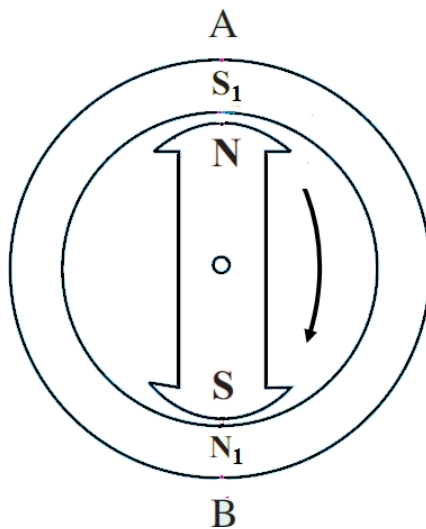


Figure 6.11: •

First of all consider the stator poles are stationary and situated at points A and B as shown in the Figure 6.10. The rotor pole N and the stator pole  $N_1$  repel each other, with the result that the rotor tends to rotate in the anticlockwise direction.

After a half period, stator poles reversed and interchange their positions but the polarity of rotor remains the same. Now stator pole  $N_1$  is at B and stator pole  $S_1$  is at A. Under these conditions, the rotor pole N and the stator pole  $S_1$  attract each other. Hence, rotor tends to rotate clockwise direction as shown in Figure 6.11. Thus the torque acting on the synchronous motor is not unidirectional but pulsating. So the *synchronous motor is not self starting*.

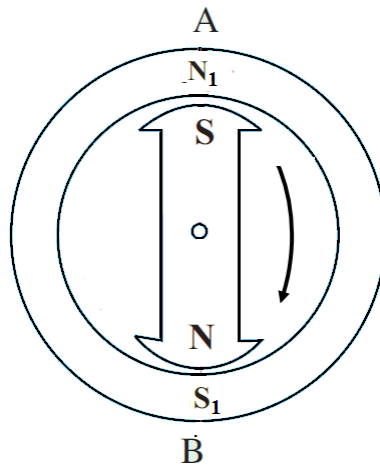


Figure 6.12: •

After the stator and rotor poles attract each other, if the rotor poles also change their positions along with the stator poles, then they will continuously experience a unidirectional torque i.e., clockwise torque, as shown in Figure 6.12

### 6.8 Methods of Starting of Synchronous Motor

Synchronous motors are not inherently self starting. In order to get synchronized, they need some external means to bring their speed close to synchronous speed. The various methods of starting synchronous motors are given below

#### 6.8.1 Using Damper Winding

In order to make a synchronous motor self-starting, special windings known as *damper windings* are provided on the rotor poles. Damper winding consisting of short circuited copper bars placed in the slots in the pole faces. The bars are short circuited with the help of end rings acts as a squirrel cage rotor winding of an induction motor. When an AC supply is given to the stator winding, rotating magnetic field is produced which causes rotor to rotate just like squirrel-cage induction motor. When the

motor attains nearly 95% of synchronous speed, DC supply is given to the field winding. At a particular instant motor gets pulled into synchronism and starts rotating at a synchronous speed.

As rotor rotates at synchronous speed, the relative motion between damper winding and the rotating magnetic field is zero. Hence when the motor is running as synchronous motor, there can not be any induced e.m.f. in the damper winding. So damper winding is not active when the machine is running at synchronous speed.

### 6.8.2 Using Pony Motor

In this method, the rotor is brought to the synchronous speed with the help of small induction motor called *pony motor*. This induction motor is capable of rotating the synchronous motor to synchronous speed. Once the rotor attains the synchronous speed, the DC excitation to the rotor is switched on. When the motor reaches synchronism, pony motor is disconnected from the supply. After the motor is synchronised, it continues to rotate as synchronous motor.

### 6.8.3 Using small DC motor

Some times a large synchronous motor can be started by means of DC compound motor coupled on it. The speed of the motor is varied by means of a speed regulator and it rotates the synchronous motor at synchronous speed. Then the excitation to the rotor is provided. Once motor starts running as a synchronous motor, the same DC machine acts as a DC generator called *exciter*. The field of the synchronous motor is then excited by this exciter itself.

## 6.9 V-curves

The variation of excitation or field current  $I_f$  causes the variation of armature current  $I_a$ . The V-curves of a synchronous motor gives the variation of armature current with its field current, when motor input is kept constant. They are so called because of their shape. Figure 6.13 shows the family of such curves for different load condition.

The power factor of the synchronous motor can be controlled by varying the field current  $I_f$ . The armature current can also be controlled by varying the field current.

Consider the input power of the machine is constant and the motor is running at no load. When the field current is increased, the armature current  $I_a$  decreases and the motor operates at lagging power factor until the armature current becomes minimum. When the armature current is minimum, the motor is operating at unity power factor.

If the field current is increased further, the armature current increases and the motor start operating as a leading power factor. The graph drawn between armature current and field current is known as V- curve.

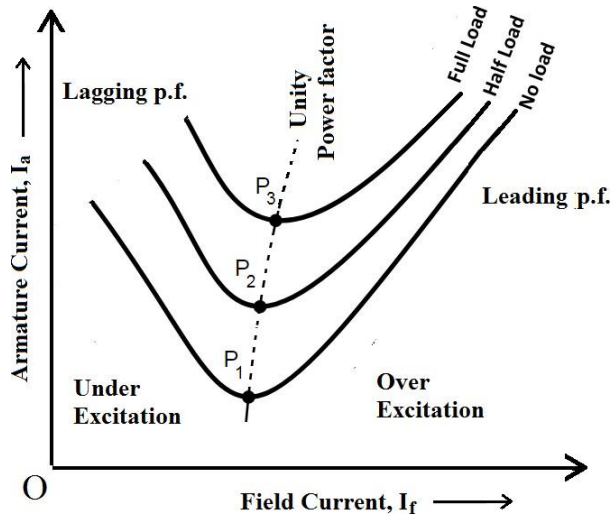


Figure 6.13: V-curves

When the armature current is minimum, the motor is operating at unity power factor. The curve connecting the Unity Power Factor points of V curves for various loads is called the *Compounding Curve*.

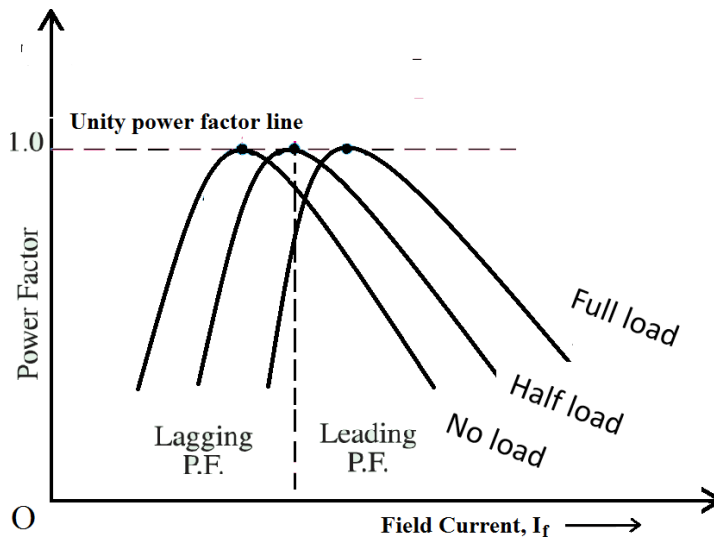


Figure 6.14: Inverted V-curves

When the field current  $I_f$  is varied, armature current also varies results in the change in power factor. Decreasing the field current below the minimum armature current results in lagging power factor. Similarly, increasing field current above the minimum armature current results in lagging power factor. The machine operates at unity power factor when the armature current is minimum. The Figure 6.14 shows

the variation of power factor with field current at different loads. This curve is known as inverted V-curve.

### 6.10 Synchronous Condenser

Synchronous Condenser is also known as Synchronous Compensator or Synchronous Phase Modifier. A synchronous condenser is an *over-excited synchronous motor* running without a mechanical load. It can generate or absorb reactive volt-ampere (VAR) by varying the excitation of its field winding. Synchronous condenser draws leading current when it operating at over-excited condition,

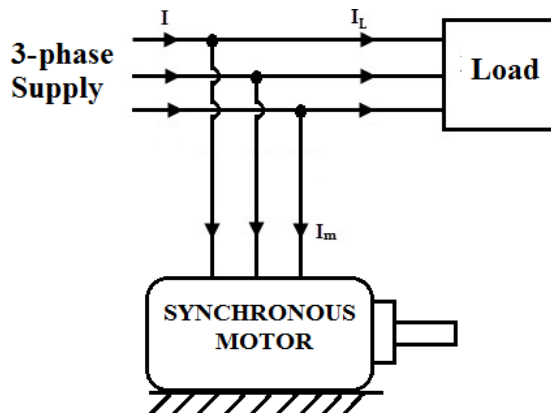


Figure 6.15: Synchronous Condenser

When the synchronous motor is running at unity power factor, the DC excitation is said to be normal. Over-excitation causes the motor to operate at a leading power factor and under-excitation of the motor causes it to operate at a lagging power factor. When the motor is operated at no load with over-excitation, it draws current that leads the voltage by nearly  $90^\circ$ . So the resultant current lags behind the voltage by a small value and thereby improving the overall power factor.

Figure 6.16 shows the phasor diagram of the system shown in Figure 6.15 in which the load current  $I_L$  lags behind the voltage  $V$  by an angle  $\phi_L$ . The current drawn by the synchronous motor  $I_m$  leads the voltage by an angle  $\phi_m$ . So the resultant of  $I_L$  and  $I_m$  gives the current  $I$  which lags behind the voltage by angle a small angle  $\phi$ . Hence  $\cos \phi$  is greater than the  $\cos \phi_L$ . So the overall power factor of the system is improved by using a synchronous condenser.

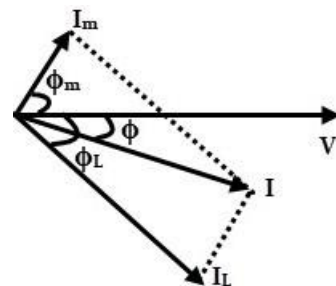


Figure 6.16: •

Both transformers and induction motors draw lagging currents from the supply line. This causes the reduction in power factor. But an over-excited synchronous motor



## Synchronous Machines

runs at leading power factor. This property of the synchronous motor helps in phase advancing and thereby improving the power factor in the case of industrial loads driven by induction motors and lighting and other loads supplied through transformers. Thus, it behaves like a capacitor and under such operating conditions, the synchronous motor is called a synchronous capacitor. Since a synchronous condenser behaves like a variable capacitor, so it is called a *synchronous capacitor* or *synchronous condenser* which used in power transmission systems to regulate line voltage.

### 6.10.1 Advantages

1. Power factor can be controlled by varying field excitation.
2. The motor windings have high thermal stability to short circuit currents.
3. It has the ability to remove fault current

### 6.10.2 Disadvantages

1. There are losses in the motor.
2. High maintenance cost
3. It produces noise.
4. Expensive than the capacitors of the same rating.

## 6.11 Applications of Synchronous Motors

1. It has higher efficiency compared to induction motor. So it can be used for constant speed applications such as fans, blowers, reciprocating pumps, compressors etc.
2. Synchronous motors are suitable for low speed application because the power factor can always be adjusted to unity and efficiency is high.
3. Overexcited synchronous motors can be used for power factor correction in substation and generating station.
4. It can be used to improve voltage regulation of transmission lines.
5. Since the speed is proportional to frequency, with the help of power electronic converters, it can be used for variable speed applications such as textile and paper industry.

**Table 6.2: Comparison Synchronous Motor and Induction Motor**

	<b>Synchronous Motor</b>	<b>Induction Motor</b>
1	Construction is complex and expensive	Construction is simple and cheap
2	It is not self-starting	It is self-starting
3	Separate DC source is required for excitation	Separate DC source is not required for excitation
4	Speed is constant irrespective of load	Speed decreases with increase in load
5	It operates at synchronous speed	It operates at speed below synchronous speed
6	Speed control is not possible	Speed control is possible
7	It operates at wide range of power factors	Operates at lagging power factor
8	It can be used as synchronous condenser for power factor correction	It cannot be used as synchronous condenser
9	Hunting is present	Hunting is absent
10	Need frequent maintenance	It is maintenance free

**Example 6. 1**

A 3-phase, 12-pole alternator has a star-connected winding with 120 slots and 10 conductors per slot. The flux per pole is 0.03 Wb, Sinusoidally distributed and the speed is 500 r.p.m. Find the phase and line e.m.f. Distribution factor=0.96. Assume full-pitched coil.

**Solution :** For full pitch coil,  $K_p = 1$

Distribution factor,  $K_d = 0.96$

$$\text{Frequency, } f = \frac{PN}{120} = \frac{12 \times 500}{120} = 50 \text{ Hz}$$

$$\text{Total number of conductors, } Z = 120 \times \frac{10}{3} = 400$$

$$\text{Number of turns, } T = 400/2 = 200/\text{phase}$$

$$\begin{aligned} \text{Phase voltage, } E_{ph} &= 4.44 K_p K_d f \Phi T \\ &= 4.44 \times 1 \times 0.96 \times 50 \times 0.03 \times 200 \\ &= 1278.72 \text{ V} \end{aligned}$$

p.f. = 0.8 leading

$$\begin{aligned}
 E_0 &= \sqrt{(V \cos \varphi + I R_a)^2 + (V \sin \varphi - I X_s)^2} \\
 &= \sqrt{(1732 \times 0.8 + 3.85)^2 + (1732 \times 0.6 - 57.56)^2} \\
 &= 1701 \text{ V} \\
 \% \text{ Regulation} &= \frac{E_0 - V}{V} \times 100 \\
 &= \frac{1701 - 1732}{1732} \times 100 \\
 &= -1.79\%
 \end{aligned}$$

### Example 6. 2

A 16-pole, 3-phase alternator has a star-connected winding with 144 slots and 10 conductors/slot. The flux/pole is 0.03 Wb, distributed sinusoidally and the speed is 375 r.p.m. Find the line voltage assuming full-pitched coils with distribution factor 0.96.

(KU ME 2015)

**Solution :**

Distribution factor,  $K_d = 0.96$

Pitch factor,  $K_p = 1$

$$\begin{aligned}
 \text{Frequency, } f &= \frac{P N}{120} \\
 &= \frac{16 \times 375}{120} \\
 &= 50 \text{ Hz}
 \end{aligned}$$

$$\text{Total number of conductors, } Z = 144 \times \frac{10}{3} = 480$$

Number of turns,  $T = 480/2 = 240/\text{phase}$

$$\begin{aligned}
 \text{Phase voltage, } E_{ph} &= 4.44 K_p K_d f \Phi T \\
 &= 4.44 \times 1 \times 0.96 \times 50 \times 0.03 \times 240 \\
 &= 1534 \text{ V}
 \end{aligned}$$

$$\begin{aligned}
 \text{Line voltage, } E_L &= \sqrt{3} E_{ph} \\
 &= \sqrt{3} \times 1534 \\
 &= 2657 \text{ V}
 \end{aligned}$$

### Example 6. 3

Calculate the no load terminal voltage at a 3 phase, 8 pole star connected alternator having the following data:

Flux per pole=55 mWb. total no. of slots in the armature=72, no. of conductors per slot=10. Distribution factor=0.96. Assume full pitch coils.

(KU ME 2016)

**Solution :**

Distribution factor,  $K_d = 0.96$

Pitch factor,  $K_p = 1$

Frequency,  $f = 50 \text{ Hz}$

Total number of conductors,  $Z = 72 \times \frac{10}{3} = 240$

Number of turns,  $T = 240/2 = 120/\text{phase}$

Phase voltage,  $E_{ph} = 4.44 K_p K_d f \Phi T$

$$= 4.44 \times 1 \times 0.96 \times 50 \times 55 \times 10^{-3} \times 120$$

$$= 1406.59 \text{ V}$$

Line voltage,  $E_L = \sqrt{3} E_{ph}$

$$= \sqrt{3} \times 1534$$

$$= 2436.29 \text{ V}$$

### Example 6. 4

A three-phase star connected alternator is rated at 1600 KVA, 13500 Volt. The armature resistance and synchronous reactance are 1.5 and 30 ohms respectively per phase. Calculate the percentage voltage regulation for a load of 1820 KW at 0.8 leading p.f.

(KTU ME 2018)

**Solution :**

Load,  $P = 1820 \text{ KW}$

Armature resistance,  $R_a = 1.5 \Omega$

Synchronous reactance,  $X_s = 30 \Omega$

$$\begin{aligned} \text{Phase Voltage, } V_{ph} &= \frac{V_L}{\sqrt{3}} \\ &= \frac{13500}{\sqrt{3}} \\ &= 7794 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Power factor, } \cos \varphi &= 0.8 \\ \sin \varphi &= \sqrt{1 - 0.8^2} \\ &= -0.6 \quad \text{sign negative for leading pf} \end{aligned}$$

$$\begin{aligned} \text{Load Current, } I &= \frac{\text{Load}}{3 V_L \cos \varphi} \\ &= \frac{1820 \times 10^3}{3 \times 13500 \times 0.8} \\ &= 97.29 \text{ A} \end{aligned}$$

Open circuit voltage perphase,

$$\begin{aligned} E_0 &= \sqrt{(V_{ph} \cos \varphi + IR_a)^2 + (V_{ph} \sin \varphi + IX_s)^2} \\ &= \sqrt{(7794 \times 0.8 + 97.29 \times 1.5)^2 + (7794 \times (-0.6) + 97.29 \times 30)^2} \\ &= 6619 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Percentage regulation} &= \frac{E_0 - V_{ph}}{V_{ph}} \times 100 \\ &= \frac{6619 - 7794}{7794} \times 100 \\ &= -15\% \end{aligned}$$

**Note : Voltage regulation for both lagging and unity power factor can also be calculated as follows:**

**Lagging Power factor**

$$\text{Phase Voltage, } V_{ph} = 7794 \text{ V}$$

$$\text{Load, } I = 97.29 \text{ A}$$

$$\begin{aligned} \text{Power factor, } \cos \varphi &= 0.8 \\ \sin \varphi &= \sqrt{1 - 0.8^2} \\ &= 0.6 \quad \text{sign positive for leading pf} \end{aligned}$$

$$\begin{aligned} E_0 &= \sqrt{(V_{ph} \cos \varphi + IR_a)^2 + (V_{ph} \sin \varphi + IX_s)^2} \\ &= \sqrt{(7794 \times 0.8 + 97.29 \times 1.5)^2 + (7794 \times 0.6 + 97.29 \times 30)^2} \\ &= 9920 \text{ V} \end{aligned}$$

## Synchronous Machines

$$\begin{aligned}\text{Percentage regulation} &= \frac{E_0 - V_{ph}}{V_{ph}} \times 100 \\ &= \frac{9920 - 7794}{7794} \times 100 \\ &= 27.28\%\end{aligned}$$

### Unity factor

Phase Voltage,  $V_{ph} = 7794 \text{ V}$

$$\begin{aligned}\text{Load Current, } I &= \frac{\text{Load}}{3 V_L \cos \varphi} \\ &= \frac{1820 \times 10^3}{3 \times 13500 \times 1} \\ &= 77.84 \text{ A}\end{aligned}$$

Power factor,  $\cos \varphi = 1$

$\sin \varphi = 0$

$$\begin{aligned}E_0 &= \sqrt{(V_{ph} \cos \varphi + IR_a)^2 + (V_{ph} \sin \varphi + IX_s)^2} \\ &= \sqrt{(7794 \times 0.8 + 97.29 \times 1.5)^2 + (7794 \times 0.6 + 97.29 \times 30)^2} \\ &= 9920 \text{ V}\end{aligned}$$

$$\begin{aligned}\text{Percentage regulation} &= \frac{E_0 - V_{ph}}{V_{ph}} \times 100 \\ &= \frac{9920 - 7794}{7794} \times 100 \\ &= 27.28\%\end{aligned}$$

## EXERCISE

1. Explain the advantage of having rotating field system rather than a rotating armature system in synchronous machines ?
2. Explain cylindrical and salient pole rotors ?
3. Sketch salient pole and non salient pole rotors ?
4. Define voltage regulation of an alternator ?
5. Explain the determination of voltage regulation by EMF method ?
6. Explain how the rotating magnetic field is developed in synchronous machines ?

7. Compare salient pole and cylindrical rotor machines ?
8. Derive an expression for the induced emf of a synchronous generator ?
9. Explain the construction of cylindrical rotor machines ?
10. Explain the principle of operation of a synchronous motor ?
11. Explain why the synchronous motor is not self starting ?
12. Explain different starting methods of synchronous motors ?
13. Explain how a synchronous machine can be used for power factor improvement ?
14. Explain the working of a synchronous condenser ?
15. What are V curves and inverted V curves ?
16. Explain compounding curve ?
17. Is it possible for a synchronous motor on no load to act as a capacitor ?
18. Find the armature conductors in series per phase required for the armature of a 3 phase, 50 cycles/sec, 10 pole alternator with 90 slots. The winding is to be star connected to give a line voltage of 11000 V. The flux/pole is 0.16 Wb ?
19. Calculate the no load line and phase voltage of a 3 phase, 12 pole star connected alternator having the following data:  
Flux per pole=0.03 Wb. total no. of slots in the armature=144, no. of conductors per slot=10. Distribution factor=0.96. Assume full pitch coils ?
20. From the following test results, determine the voltage regulation of a 22-KV, 3-phase alternator delivering a current of 100 A at (i) unity p.f. (ii) 0.8 leading p.f. and (iii) 0.71 lagging p.f.  
Test results : Full-load current of 80 A is produced on short-circuit by a field excitation of 3 A. An e.m.f. of 500 V is produced on open-circuit by the same excitation. The armature resistance is  $0.8\Omega$  ?

# Special Machines

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Special electrical machines are another category of electrical machines which are designed for specific applications. It gained much popularity and its applications are increasing day by day. Special electrical machines include stepper motors, reluctance motors, permanent magnet motors, brush less DC motors, linear motors, etc. Although special electrical machines were invented along with conventional electrical machines, their application were limited due to the unavailability of proper control strategies. Now due to the advancement of power electronics and control system they are now commonly seen in many domestic and industrial applications.

## Stepper Motor

Stepper motors is a special electrical machines which rotates in discrete angular steps in response to each input current pulse received by its controller. Each step moves the shaft in a fixed angle. It is also called stepping motors or step motors. Stator has multiple coils that are organized in groups called *phases* which are energised using current pulses. Rotor has no electrical connection, but has salient poles which may or may not have permanent magnet. When each phase is energised in sequence using current pulse, the motor will rotate, one step at a time. So a stepper motor converts a train of input pulses to a train of step movements. The angle through which the the motor shaft rotates for each current pulse is called the *step angle*.



Due to the advancement of computers, there have been wide spread demand of stepper motors. Using a computer controlled stepping you can achieve very precise



position control and speed control without using closed loop control. They can also be controlled directly by microprocessors and programmable controllers. Stepper motors are ideally suited for applications where either precise positioning or precise speed control or both are required in automation systems. It has the ability to control an open loop system. So it requires no feedback connection and expensive sensing and feedback devices such as *optical encoders*.

### 7.2 Uses of Stepper Motor

1. **Position Control** : Stepper motor can be used in applications requiring precise position control such as 3D printers, office machines, X-ray machine, CNC machines, Camera platforms and X,Y Plotters. Some disk drives also use stepper motors for positioning the read/write head.
2. **Speed Control** : It can also be used for precise speed control applications such as process automation and robotics.
3. **Low Speed Torque** : Stepper motors, with their ability to produce high torque at a low speed while minimizing vibration, are ideal for applications requiring quick positioning over a short distance.

### 7.3 Advantages of Stepper Motor

1. Construction is simple.
2. Less expensive
3. It operates at full torque at standstill.
4. Less maintenance is required
5. Higher accuracy in position control
6. A wide range of rotational speeds can be achieved
7. It uses open-loop control which makes it simpler and less costly to control.
8. Rapid response to start, stop and reverse.
9. Very reliable since there are no contact brushes in the motor.

### 7.4 Disadvantages of Stepper Motor

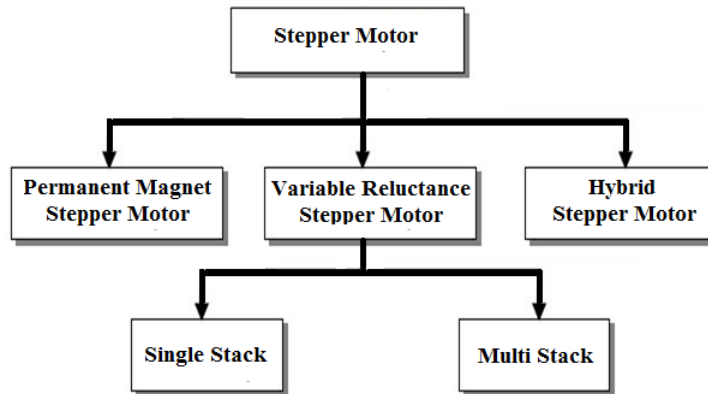
1. It draws more current as compared to the DC motor.
2. Torque decreases when the speed increases
3. Less efficiency.

4. The Resonance condition arises if not controlled properly and requires micro stepping.
5. Control is difficult at higher speed
6. They do not have integral feedback for position.

### 7.5 Types of Stepper Motor

Stepper motor can be classified into several types according to the machine structure and principle of operation. There are three main types of stepper motors, they are:

1. Variable Reluctance stepper Motor
2. Permanent Magnet stepper Motor
3. Hybrid Stepper Motor



#### 7.5.1 Variable Reluctance Stepper Motor

Figure 7.1 shows variable reluctance stepper motor. It consists of a soft iron multi-toothed rotor and a wound stator. The stator windings are energized with DC current. Direction of rotation of motor is independent of the polarity of the stator. It can be classified into single stack type multi-stack type. Stator has salient poles. Rotor rotates when the rotor teeth are attracted to the energized stator poles. A rotor step takes place when one stator phase is de-energized and the next phase in sequence is energized, thus creating a new position of minimum reluctance for the rotor. So the electrical pulses applied to the stator windings produce alternating magnetic fluxes over the rotor teeth which causes a twisting effect. The reluctance of the magnetic circuit of rotor and stator teeth varies with respect to the angular position of the rotor. It is rarely used in industry due to less detent torque.

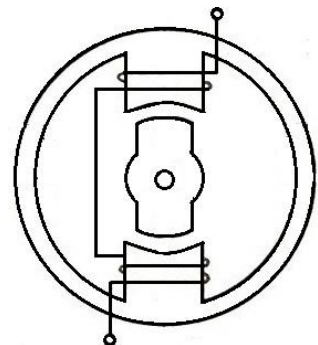


Figure 7.1:

## Advantages

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1. High torque to inertia ratio
2. Robust
3. Higher acceleration.
4. Fast dynamic response
5. High stepping rate and speed slewing capability.
6. Efficient cooling possible

## Disadvantages

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1. Vibrations
2. Complex control circuit
3. No smaller step angle
4. No detent torque.

### 7.5.2 Permanent Magnet Stepper Motor

Figure 7.2 shows permanent magnet stepper motor. It consists of a permanent magnet in the rotor and a wound stator. The stator windings are energized with DC current but its rotor poles are permanently magnetized. Direction of rotation depends upon the polarity of the stator current. Stator has salient poles. The rotor no longer has teeth like variable reluctance stepper motor. Rotor has alternate north and south poles situated in a straight line parallel to the rotor shaft. The electrical pulses applied to the stator windings produce alternating magnetic field which interact with the permanent magnets of the rotor. This results in high torque level which makes it more efficient. Since the rotor consists of permanent magnet, it provide an improved magnetic flux intensity which helps it to exhibits improved torque-speed characteristics when compared with the variable reluctance motor. This motor is an ideal choice for application requires very low speed such as line printer.

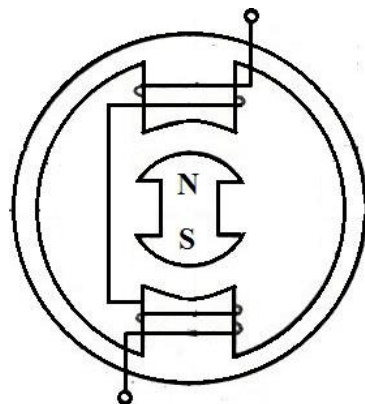


Figure 7.2:

### Advantages

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1. Simple construction
2. Produce Detent torque
3. Higher holding torque
4. Better damping
5. Less maintenance required

### Disadvantages

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1. High inertia
2. More weight
3. Bigger step angle
4. Fixed rated torque.
5. Limited power output and size
6. Possibility of reduction in magnetic strength which affects the performance of motor.

### 7.5.3 Hybrid Stepper Motor

Hybrid stepper motor is an improved version of single stack variable reluctance stepper motor. It combines the best properties of variable reluctance and permanent magnet stepper motors to achieve maximum performance. The hybrid stepper motor is more expensive than the permanent magnet stepper motor but shows better performance with respect to step resolution, torque, and speed. Typical step angles for the hybrid stepper motor ranges from  $3.6^\circ$  to  $90^\circ$ . The rotor of a hybrid stepper is multi-toothed which provides better path for the distribution of magnetic field in the air gap. This further increases the detent, holding and dynamic torque characteristics of the motor when compared with both the variable reluctance and permanent magnet stepper motors. This motor type has some advantages such as low inertia. The direction of rotation of hybrid stepper motor depends on the polarity of the stator current. The most commonly

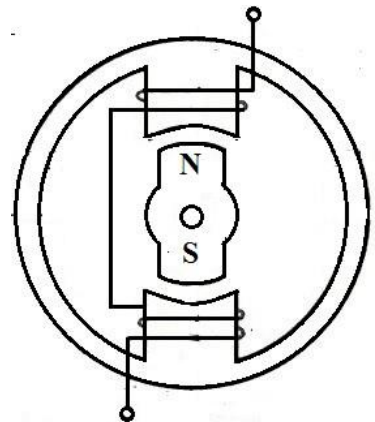


Figure 7.3: •

used types of stepper motors are the hybrid and permanent magnet stepper motors. For low cost application permanent magnet is a good choice but for precise control hybrid stepper motor is the best choice.

### Advantages

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1. Step angle error is very small and non-cumulative.
2. Excellent response to starting, stopping and reversing.
3. produce detent torque
4. Less tendency to resonate
5. Small step angle possible
6. High torque per package size.
7. Holding torque at standstill.

### Disadvantages

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1. High inertia
2. More weight
3. Possibility of resonance
4. Possibility of vibration

## 7.6 Step angle

Step angle is defined as the angle through which the rotor of a stepper motor moves when one command pulse is received at the input of the stator. It is denoted by  $\beta$ . The angular position of a stepper motor is decided by the step angle and is expressed in degrees. When the step angle is small, the number of steps per revolution becomes large and the resolution or accuracy of positioning obtained. A standard motor will have a step angle of 1.8 degrees with 200 steps per revolution. The various step angles like  $90^\circ$ ,  $45^\circ$ ,  $15^\circ$ ,  $1.8^\circ$ ,  $2.5^\circ$ ,  $7.5^\circ$  are common in simple motors.

If the rotor and stator poles are  $N_r$  and  $N_s$  respectively, then the step angle is given by the expression:

$$\text{Step angle, } \beta = \frac{(N_s - N_r)}{N_s \cdot N_r} \times 360^\circ \quad (7.1)$$

The step angle can also be determined using the number of stator phases ( $m$ ) and the number of rotor teeth ( $N_r$ ) by the expression

$$\text{Step angle, } \beta = \frac{360}{m.N_r} \quad (7.2)$$

The accuracy of positioning by stepper motor depends on the resolution. Higher the resolution greater will be the accuracy. Resolution is the number of steps required to complete one revolution of the rotor shaft.

$$\begin{aligned} \text{Resolution, } &= \frac{\text{Number of steps}}{\text{Number of revolutions of rotor}} \\ &= \frac{360^\circ}{\beta} \end{aligned} \quad (7.3)$$

### Example 7. 1

A stepper motor has a step angle of  $2.4^\circ$ . Find the resolution and the number of steps required to cause 100 revolutions. Also find the shaft speed, if the stepping frequency is 1800 pulse/sec.

#### Solution:

$$\begin{aligned} \text{Step angle, } \beta &= 2.4^\circ \\ \text{Resolution, } &= \frac{360^\circ}{\beta} \\ &= \frac{360^\circ}{2.4^\circ} \\ &= 150 \text{ steps/rev} \end{aligned}$$

$$\text{Number of steps for 100 revolutions, } = 150 \times 100$$

$$\begin{aligned} &= 15000 \\ \text{Shaft speed, } n &= \frac{\beta \times f}{360^\circ} \\ &= \frac{2.4 \times 1800}{360^\circ} \\ &= 12 \text{ r.p.s} = 720 \text{ r.p.m.} \end{aligned}$$

### Example 7. 2

A stepper motor is designed to have three phase and six poles. If it has 10 rotor teeth. Find its resolution.

**Solution:**

Number of stator phases,  $m = 3$

Number of rotor teeth,  $N_r = 10$

$$\begin{aligned}\text{Step angle, } \beta &= \frac{360^\circ}{m \times N_r} \\ &= \frac{360^\circ}{3 \times 10} \\ &= 12^\circ\end{aligned}$$

$$\begin{aligned}\text{Resolution} &= \frac{360^\circ}{\beta} \\ &= \frac{360^\circ}{12^\circ} \\ &= 30 \text{ steps/rev}\end{aligned}$$

**Example 7. 3**

If a stepper motor has 6 stator poles and having 30 stator teeth. If rotor has 20 teeth, calculate the stepping angle and resolution.

**Solution:**

Number of stator teeth,  $N_s = 30$

Number of rotor teeth,  $N_r = 20$

$$\begin{aligned}\text{Step angle, } \beta &= \frac{(N_s - N_r)}{N_s \cdot N_r} \times 360^\circ \\ &= \frac{(30 - 20)}{30 \times 20} \times 360^\circ \\ &= 6^\circ\end{aligned}$$

$$\begin{aligned}\text{Resolution} &= \frac{360^\circ}{\beta} \\ &= \frac{360^\circ}{6^\circ} \\ &= 60 \text{ steps/rev}\end{aligned}$$

**7.7 Single Stack Variable Reluctance Stepper Motor**

Single stack variable reluctance stepper motor has a laminated silicon steel stator. It has projecting poles. Stator poles carries a concentric windings. The field windings

of opposite poles are connected in series. The two coils are connected in such a way that their magnetic flux gets added, then the two coils form a *phase*. Both stator and rotor is made up of materials having high permeability. The rotor also consists of laminated projecting poles. Rotor has no winding and there is no electrical connection in rotor. In order to have self-starting capability and bi-directional rotation, the number of stator and rotor poles are not equal.

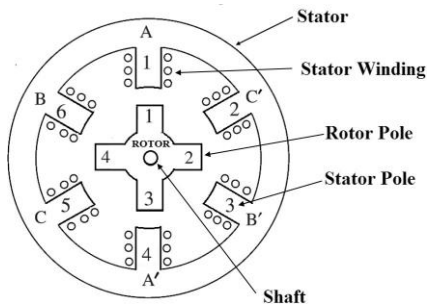


Figure 7.4: •

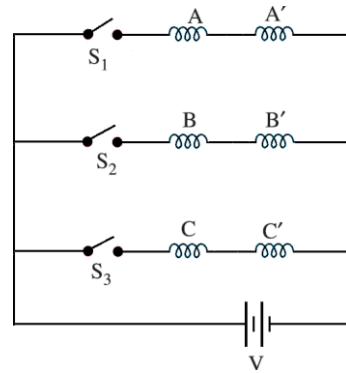


Figure 7.5: •

Figure 7.4 shows a three phase, six pole variable reluctance stepper motor. It consists of three phases, A, B and C and each phase is energised by passing electric pulse. Electrical connection of variable reluctance stepper motor as shown figure 7.5. Coil A and A' are connected in series forms a phase winding. Similarly, coil B and B' are connected in series and coils C and C' series form phase windings. When current passed through the phase, one pole becomes a N-pole and the other one becomes a S-pole. These phases are energised using a DC source. The DC source is connected to phase A , phase B and phase C through switches S<sub>1</sub>, S<sub>2</sub> and S<sub>3</sub> respectively.

### 7.7.1 Modes of Operation

It works on the principle of variable reluctance. The principle of operation of VR stepper motor explained by referring the various modes of operation given below:

#### MODE 1 : Full step operation

In this mode of operation, only one phase of stepper motor is energized at any time. When the switch S<sub>1</sub> is on, current is applied to the phase A and the coils of phase 1 and 4 are excited, the reluctance torque causes the rotor to align with the axis of phase A. The rotor teeth 1 and 3 are attracted to the stator teeth 1 and 4 respectively as shown in Figure 7.6(a). Then angle  $\theta = 0^\circ$  and the rotor cannot move until another phase is energized.

Next, the switch S<sub>2</sub> is turned on and switch S<sub>1</sub> is turned off. Now the phase A is de-energised and the phase B is energised so that the stator teeth 3 and 6 are



energised. The stator teeth 2 and 4 would align with rotor teeth 3 and 6 respectively

as shown in Figure 7.6(b). The rotor rotates in the clockwise (CW) direction and makes an angular displacement of  $30^\circ$ . The rotor gets attracted to the stator poles until the switch  $S_2$  is turned off. Table 7.1 shows the truth table of full step operation.

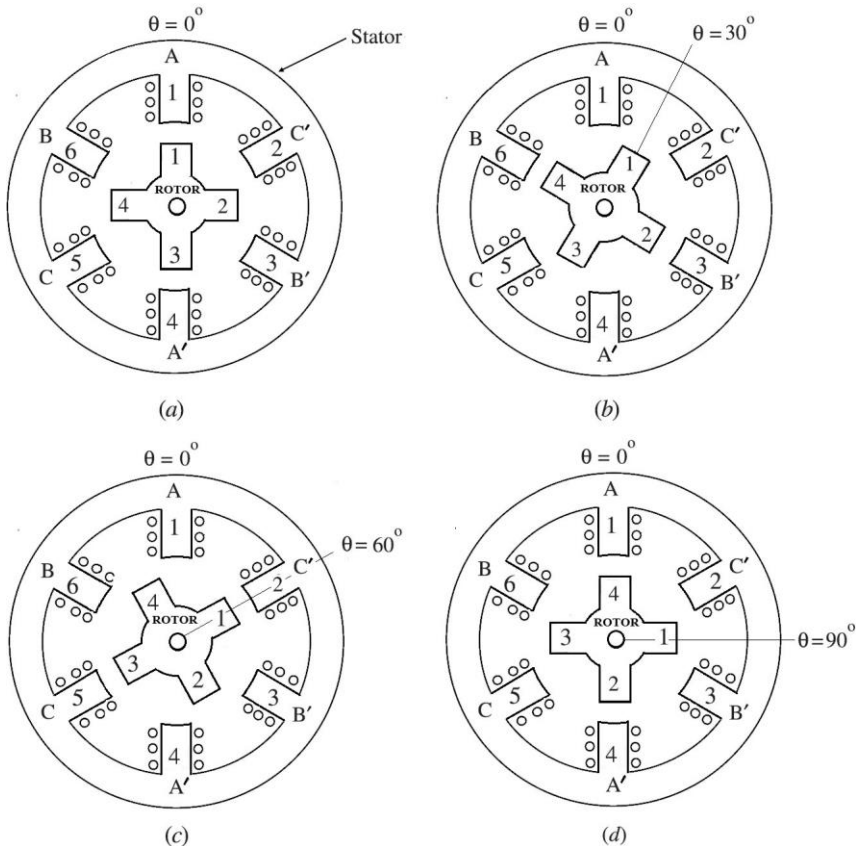


Figure 7.6: •

**Table 7.1: Truth Table**

Phase A	Phase B	Phase C	$\theta$
+	0	0	$0^\circ$
0	+	0	$30^\circ$
0	0	+	$60^\circ$
+	0	0	$90^\circ$

Next, the phase C is energised by turning on  $S_3$  and the phase B is de-energised by turning off  $S_2$ . Now the stator teeth 2 and 5 are energised. The stator teeth 2 and 5 would align with rotor teeth 1 and 3 respectively as shown in Figure 7.6(c). The rotor rotates in the clockwise (CW) direction and makes an angle of  $30^\circ$ . So a total angular displacement of  $60^\circ$  is obtained in the clockwise direction.

Now again the switch  $S_1$  is turned on and the switch  $S_3$  is turned off. Now the stator teeth 1 and 4 are energised. The stator teeth 1 and 4 would align with rotor teeth 4 and 2 respectively as shown in Figure 7.6(d). So if we continuously closing the switches in the sequence  $S_1$ - $S_2$ - $S_3$ - $S_1$  and thus energizing stator phases in sequence A-B-C-A , the rotor will rotate in the clockwise direction with a step angle of  $30^\circ$ . Also if we continuously closing the switches in the sequence  $S_3$ - $S_2$ - $S_1$ - $S_3$  and thus energizing stator phases in sequence C-B-A-C , the rotor will rotate in the anti-clockwise direction with a step angle of  $30^\circ$ . This operation of stepper motor is called full step operation.

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### **MODE 2 : Two-phase-ON Mode**

In this mode of operation, two stator phases are energised simultaneously. The main advantage of this type of operation is that the holding torque developed by the stepper motor is more and better damped response is obtained compared to full step operation. Table 7.2 gives the truth table. When the switches  $S_1$  and  $S_2$ , the phases A and B are energized together and the rotor experiences attraction from both phases. Due to the torque from phase A and phase B, the rotor comes to rest in a point mid-way between the two adjacent full step position. The rotor will rotate in the clockwise direction with an angle of  $15^\circ$ .

**Table 7.2: Truth Table**

Phase A	Phase B	Phase C	$\theta$
+	+	0	$15^\circ$
0	+	+	$45^\circ$
+	0	+	$75^\circ$
+	+	0	$105^\circ$
0	+	+	$135^\circ$
+	0	+	$165^\circ$
+	+	0	$195^\circ$

If the stator phases are switched in the sequence AB-BC-CA-AB etc., the motor will take full steps of  $30^\circ$  each in the clockwise direction. If the stator phases are switched in the sequence AC-CB-BA-AC etc., the motor will take full steps of  $30^\circ$  each in the anti-clockwise direction.

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### **MODE 3 : Half-step Operation**

This method is the combination of one phase ON and two phase ON mode of operations. In this mode of operation, the step angle obtained is half of the full step angle.

Half-step operation is obtained by exciting the three phases in the sequence A-AB-B-BC-C-CA etc. So method uses 1-phase-ON and 2-phase-ON modes alternately.

**Table 7.3: Truth Table**

Phase A	Phase B	Phase C	$\theta$
+	0	0	$0^\circ$
+	+	0	$15^\circ$
0	+	0	$30^\circ$
0	+	+	$45^\circ$
0	0	+	$60^\circ$
+	0	+	$75^\circ$
+	0	0	$90^\circ$

This method is also known as wave excitation and it causes the rotor to advance in steps of  $15^\circ$ . The truth table for the phase pulsing sequence in half-stepping is shown in Table 7.3.

#### **MODE 4 : Micro stepping**

In Micro stepping, the currents in the stator windings are continuously varied to break up one full step into many smaller discrete steps. It is also known as mini-stepping. In this method, two phases are energised simultaneously as in 2-phase-ON mode but with the two currents deliberately made unequal. The current in one phase is held constant while the current in the other phase is increased in very small increments upto the maximum current. After that the current in the first phase is reduced to zero using the same very small increments. The advantages of microstepping in stepper motor are Improvement in resolution, DC motor like performance, elimination of resonance and Rapid motion at micro stepping rate.

#### **7.8 Multi Stack Variable Reluctance Stepper Motor**

The multi-stack variable reluctance stepper motors are used to obtain smaller step angles, typically in the range of  $2^\circ$  to  $15^\circ$ . Although three stacks are common, a multi stack motor may employ as many as seven stacks. It has higher torque to volume ratio and efficiency compared to single stack motor.

The stator and rotor of multi-stack motor is divided along its axial length into a number of sections or stacks. All the stacks are magnetically isolated from others. Each phase has separate stator and rotor stacks. All the coils of stator stacks are connected in series and it can be excited to make north and south poles. Both stator and rotor have the same number of poles. Usually stator poles have sub teeth.

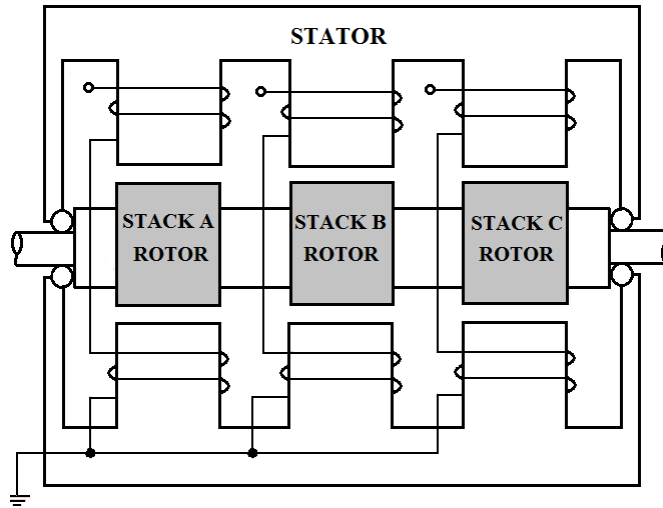


Figure 7.7: Multi Stack Variable Reluctance Stepper Motor

Figure 7.7 shows a three stack variable reluctance stepper motor. The stators and rotors have the same number of poles and therefore same pole pitch. For a multi stack motor with  $m$  number of phases, the stator poles in all  $m$  stacks are aligned, but the rotor poles are displaced by  $1/m$  of the pole pitch. All the stator pole windings in a given stack are excited simultaneously and, therefore the stator winding of each stack forms one phase. So the motor has the same number of phases as number of stacks.

If there are 12 stator and rotor poles in each stack. The pole pitch for the 12 pole rotor is  $30^\circ$ , and the step angle or the rotor pole teeth are displaced by  $10^\circ$  from each other.

$$\text{Step angle} = \frac{360^\circ}{3 \times 12} = 30^\circ \tag{7.4}$$

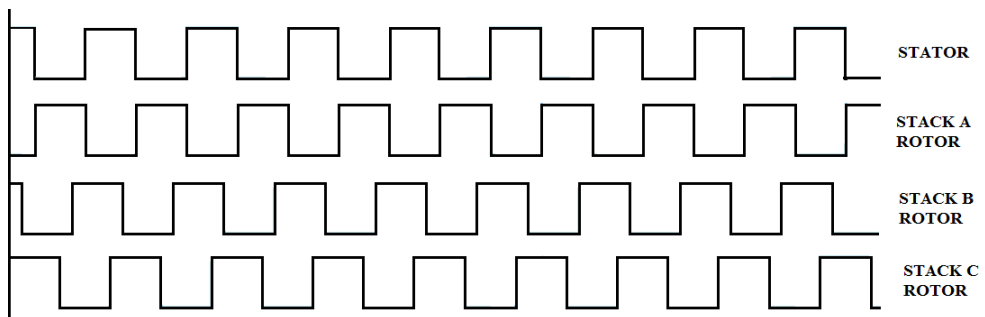


Figure 7.8: •

When the phase winding A is energised, the rotor pole of stack A are aligned with the stator poles as shown in the Figure 7.8. When phase A is de-energized,

and phase B is energised, rotor teeth of the stack B are aligned with the stator poles as shown in Figure 7.9 and the rotor rotates at an angle  $10^\circ$  in the anticlockwise direction.

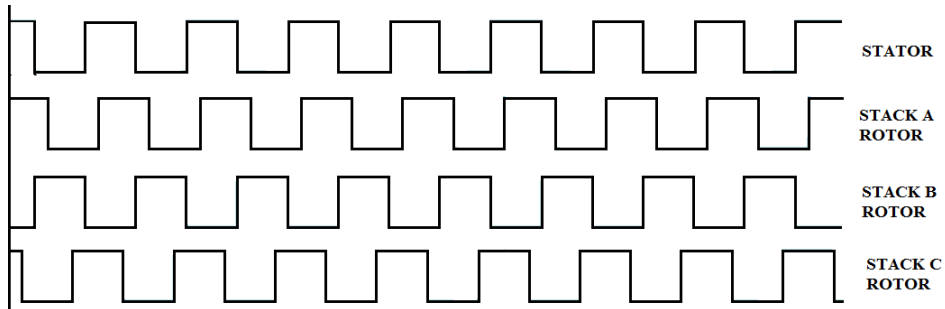


Figure 7.9: •

Similarly when phase B is de-energised, and phase C is energised, rotor poles of the stack C are aligned with the stator poles and the rotor moves another  $10^\circ$  in the anti-clockwise direction. Again, another change in the excitation of the rotor takes place, and the stator and rotor teeth align it with stack A. If we energise the stator in the order A-B-C-A etc. we will get a continuous rotation in the anti-clockwise direction.

### 7.9 Permanent Magnet Stepper Motor

The stator construction of Permanent magnet stepper motor is similar to VR stepper motor. The rotor is made of a permanent-magnet. As shown in the Figure 7.10 the stator has salient poles but the rotor is cylindrical and has radially magnetized permanent magnets. Since the rotor is made of permanent magnet, it has detent torque.

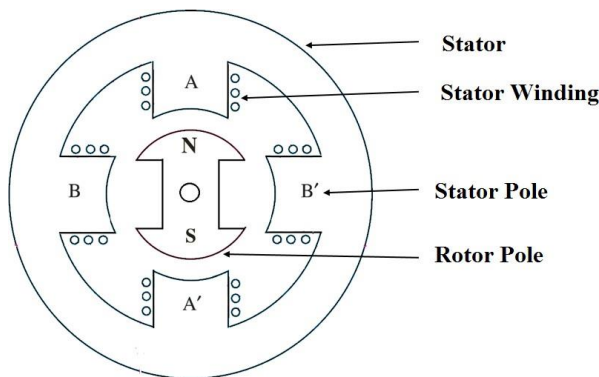


Figure 7.10: PM Stepper Motor

Figure 7.10 shows PM stepper motor with the rotor has two poles and the stator

has four poles. Since two stator poles are energized by a single winding, the motor has two phases marked A and B.

### 7.9.1 Modes of Operation

Consider a PM stepper motor having four stator poles and two rotor poles. When a particular stator phase is energized, the rotor magnetic poles move into alignment with the excited stator poles. The principle of operation of PM stepper motor explained by referring the various modes of operation given below:

#### MODE 1 : Single Phase ON Mode

The stator windings A and B can be excited with either positive or negative current. Initially phase A is energised using a positive current. Here, the S pole formed by the stator phase A and the N pole of the rotor gets locked and angle  $\theta = 90^\circ$  as shown in Figure 7.11 (a).

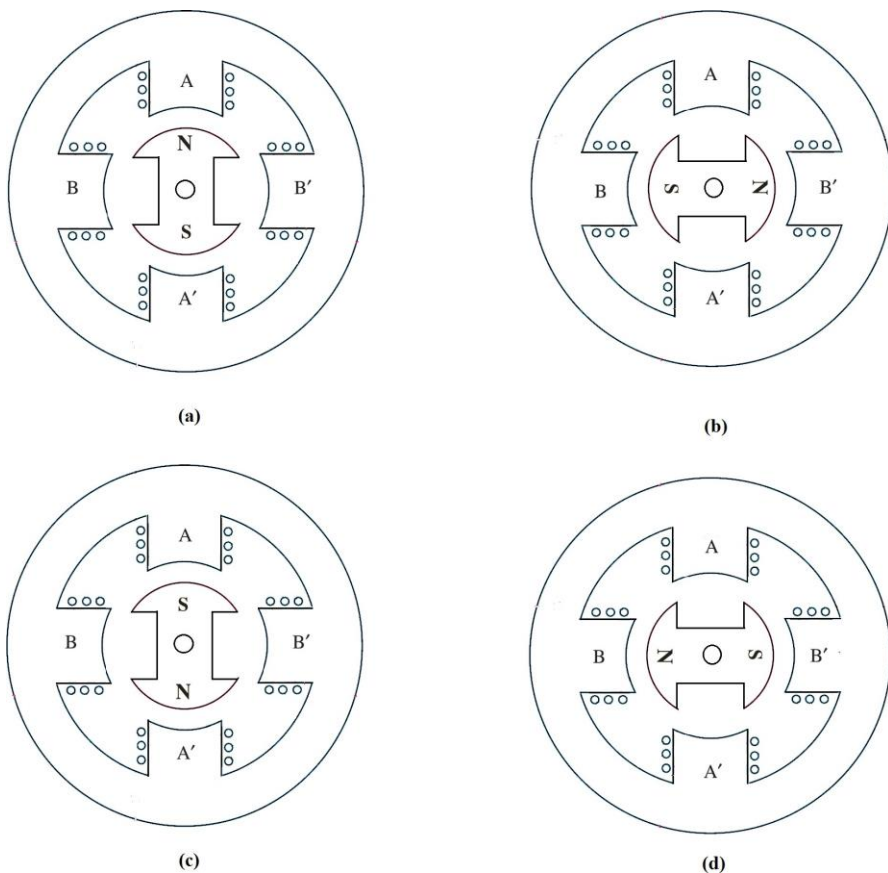


Figure 7.11: PM Stepper Motor

If positive current is given to phase B as in Figure 7.11 (b), the rotor rotates by a full step of  $90^\circ$  in the clockwise direction. Next, when phase A is excited with negative current, the rotor turns through another  $90^\circ$  in CW direction as shown in Figure 7.11 (c).

Similarly, if phase B is excited with a negative current, the rotor moves again by  $90^\circ$  further turns the rotor through another  $90^\circ$  in the same direction as shown in Figure 7.11 (d). Next, if the phase A is excited again with a positive current, makes the rotor turn through another  $90^\circ$ . The truth table is shown in Table 7.4

**Table 7.4: Truth Table**

Phase A	Phase B	$\theta$
+	0	$0^\circ$
0	+	$90^\circ$
-	0	$180^\circ$
0	-	$270^\circ$
+	0	$360^\circ$ or $0^\circ$

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### MODE 2 : Two Phase ON Mode

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In this mode of operation, two stator phases are energised simultaneously. If positive current is given to both phase A and phase B, the rotor rotates by a full step of  $45^\circ$  in the clockwise direction. Next, when phase A is excited with negative current and phase B is excited with positive current, the rotor turns through another  $90^\circ$  in CW direction and total  $135^\circ$  is obtained. Similarly, if both phase A and phase B are excited with negative current, the rotor turns through another  $90^\circ$  in CW direction. Again if phase A is excited with positive current and phase B is excited with negative current, the rotor turns through another  $90^\circ$  in CW direction is obtained. Next, both phase A and phase B are excited with positive current, the rotor turn through another  $90^\circ$ . Truth table for different possible current sequences for producing clockwise rotation are given in Table 7.5

**Table 7.5: Truth Table**

Phase A	Phase B	$\theta$
+	+	$45^\circ$
-	+	$135^\circ$
-	-	$225^\circ$
+	-	$315^\circ$
+	+	$45^\circ$



**MODE 3 : Alternate One Phase and Two Phase ON Mode**

This method is the combination of one phase ON and two phase ON mode of operations. In this mode of operation, the step angle  $45^\circ$  obtained. Truth table for different possible current sequences for producing clockwise rotation are given in Table 7.6

**Table 7.6: Truth Table**

Phase A	Phase B	$\theta$
+	0	$0^\circ$
+	+	$45^\circ$
0	+	$90^\circ$
-	+	$135^\circ$
-	0	$180^\circ$
-	-	$225^\circ$
0	-	$270^\circ$
+	-	$315^\circ$
+	0	$360^\circ$

**7.10 Linear Stepper Motor**

Linear motors works on the same basic principle as rotating motors. The difference, of course, is that linear motion is produced, rather than rotation. Linear step motors are ideal for open loop position control, velocity control and acceleration control applications with light loads. It can be integrated with a linear encoder for closed loop control.

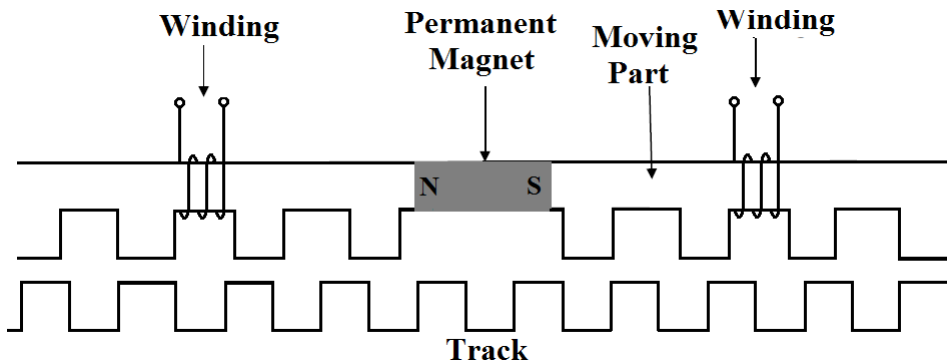


Figure 7.12: Cross Section of Linear Stepper Motor

The linear stepper motor consists of a stationary track, and a moving assembly. The moving assembly of a linear stepper motor has a number of teeth that are similar

to those found on the rotor in a conventional stepper motor, and it has two sets of field windings and a permanent magnets. The stationary track can be considered as the stator of a conventional stepper motor.

When the field windings of the moving assembly is energised, one set of teeth is aligned with the teeth of the stationary track as shown in Figure 7.12. It should be noted that the stationary track and moving assembly have similar teeth structure. A small offset is provided in order to facilitate the moving assembly to be attracted to the next available magnetic field in the track.

The magnetic flux from the electromagnets aids the flux lines of one of the permanent magnets and cancels the flux lines of the other permanent magnet. When current flow to the coil is stopped, the moving assembly will align itself to the appropriate tooth set, and a holding force ensures that there is no movement. The linear stepper motor is provided with a controller which selects the way in which the field windings are energised, so that the motor can move in the desired direction either forward or backward.

### 7.10.1 Advantages

1. Precision open-loop control
2. Simple in construction
3. Low cost position, velocity and acceleration control
4. Reliability
5. Suited for applications where space is limited
6. No servo tuning required

### 7.10.2 Applications

1. Pick and place equipment
2. Hexapod robot
3. Pumps
4. Conveyors
5. Laboratory equipment
6. Valve actuators

### 7.11 Comparison of Stepper Motors

So far we have discussed about different types of stepper motors.. For the selection stepper motor for a particular application, we need to consider various characteristics of each motor like step size detent torque, and the rotor inertia and cost.

- B Cost of hybrid stepper motor is high compared to variable reluctance and permanent magnet stepper motors. Permanent magnet stepper motor is least expensive.
- B Construction of hybrid stepper motor is complex but the construction of variable reluctance motor is simple. Complexity of construction is moderate in the case of permanent magnet stepper motor.
- B Variable reluctance stepper motors has less weight compared to hybrid motors which makes it quick response to command signal.
- B Due to low inertia, variable reluctance motor has higher allowable frequency of operation.
- B No matter what kind of excitation is, the operation of variable reluctance stepper motor is noisy compared to permanent magnet and hybrid stepper motors.
- B Hybrid stepper motors are available with smaller step sizes than variable reluctance motors. The larger step size of variable-reluctance motors, is more suited to high speed applications.
- B Hybrid stepper motor is subjected to mechanical resonance of the drive
- B Hybrid motor will produce a continuous detent torque and there is no cumulative position error. So it is more suitable for open loop operations.
- B Variable reluctance stepper motor is less pronounced to torque drop at high speed.
- B While a linear motion can be obtained by the combination of a ball screw with any type of stepper motor, giving a low cost linear actuator, the linear stepper motor has a number of performance advantages. However, it should be noted that as with any linear motor, vertical operation can prove problematic.

### 7.12 Static Characteristics of Stepper Motor

The stepper motor is excited and brought into rest or equilibrium position by supplying a current. If an external torque is applied to the shaft, an angular displacement

will occur. The relation between the external torque and the displacement is shown in Figure . This curve is called Torque-Angle characteristics and the maximum of static torque is termed the *holding torque* which occurs at  $\theta = \theta_m$ . The holding

torque is defined as the maximum static torque that can be applied to the shaft of an excited motor without causing continuous motion.

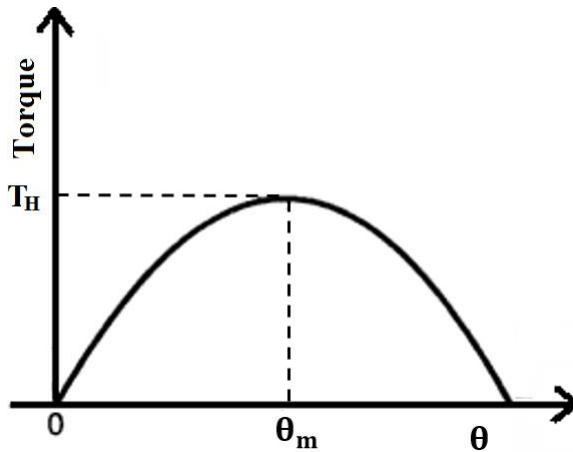


Figure 7.13: Torque-Angle Characteristics

A typical torque-current characteristics for variable reluctance and hybrid stepper motors is shown in Figure 7.14. It is seen the curve is initially linear. Torque constant of the stepper is defined as the initial slope of the torque-current curve of the stepper motor. It is also known as torque sensitivity.

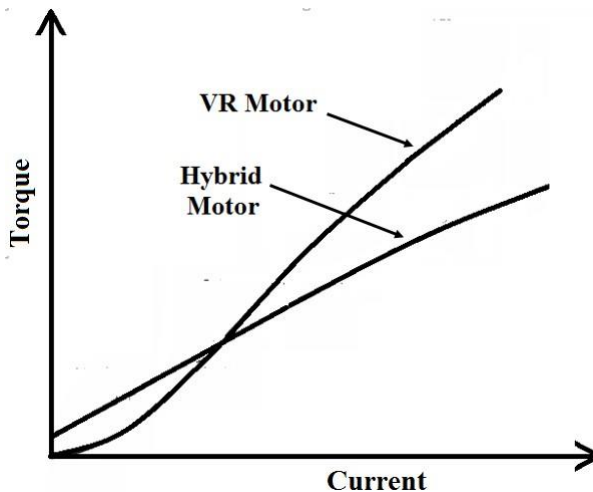


Figure 7.14: Torque-Current Characteristics

The maximum static torque appearing in the hybrid motor with no current is the *detent torque*, which is defined as the maximum static torque that can be applied to the shaft of an unexcited motor without causing continuous rotation. Detent torque is due to magnetism, and is therefore available only in permanent magnet and hybrid stepper motor.

### 7.13 Dynamic Characteristics of Stepper Motor

The characteristics relating to motors which are in motion or about to start are called dynamic characteristics. Pull-in torque and Pull-out characteristics of a stepper motor is called dynamic characteristics or torque-speed characteristics. The area between pull-in torque and pull-out torque represents those values of torque and stepping rate which the motor follows without losing synchronism provided that it has already started and synchronised. This area is known as *slew range*.

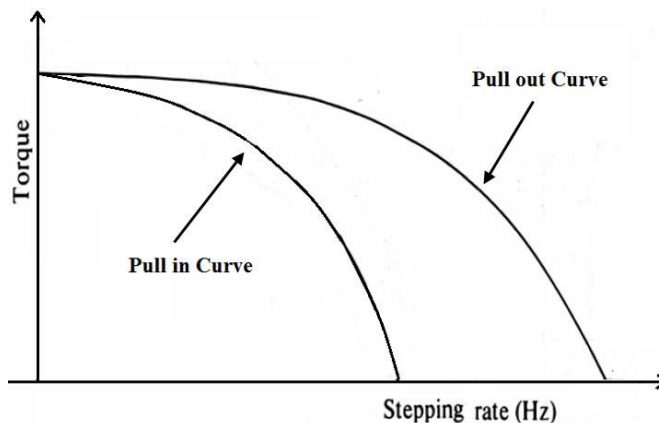


Figure 7.15: Torque-Speed Characteristics

#### 7.13.1 Pull-in Characteristics (Start-Stop)

It is also known as starting characteristics of a stepper motor. It shows the range of frictional load torque at which the motor can start and stop without losing any steps. It is also known as the characteristics of start-stop mode of operation. In start-stop mode of operation, at any operating point, the motor can start and stop without losing synchronism. In this mode, the second pulse is given to the stepper motor only after the rotor attained a steady or rest position due to first pulse. The region of start-stop mode of operation depends on the operation depends on the torque developed and the stepping rate or stepping frequency of stepper motor.

#### 7.13.2 Pull-out Characteristics (Slewing)

This is also known as the *slewing characteristic*. It gives the relation between the frictional load torque and the maximum pulse frequency with which the stepper motor can be synchronized. From Pull-out characteristics, the maximum torque that can be developed at any speed can be obtained. If the load torque is more than the motor torque, the motor will pull out of synchronism with the magnetic field, and it will stop.

In slewing mode of operation, to attain an operating point in the slewing mode without losing synchronism, at first the motor is to operate at a point in the start-stop mode and then stepping rate is increased to operate in slewing mode. Similarly to stop the motor operating in slewing mode, first the motor is to be brought to the start stop mode and then stop.

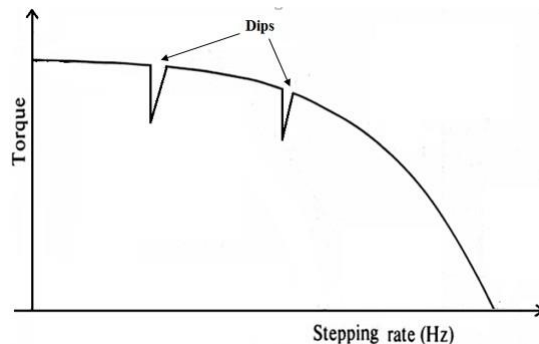


Figure 7.16: •

In practice, the the pull-out-torque characteristics shows severe dips around certain some speeds. These dips are caused by resonance between the motor and the excitation frequency. The dips in the curve corresponds to *resonance points*. For getting torque through out the speed range, the torque should be maintained less than maximum torque

### 7.13.3 Important Terms

#### Positional error

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It is the total percentage error during one complete rotation of the rotor of a stepper motor.

#### Stepping rate

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The number of steps completed per second by the rotor of a stepper motor is called stepping rate or stepping frequency.

#### Detent position

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It is the position of the rotor of an unexcited stepper motor

#### Hold position

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It is the resting position of the rotor of an excited stepper motor

### **Pull in torque**

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It is the maximum torque developed by the stepper motor for a given stepping rate in the start-stop mode of operation without losing synchronism. It is also known as limiting torque.

### **Pull out torque**

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It is the maximum torque developed by the stepper motor for a given stepping rate in the slewing mode without losing synchronism. It is also known as critical torque.

### **Pull in range**

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It is the maximum stepping rate at which the stepper motor can operate in start-stop mode developing a specific torque without losing synchronism

### **Pull out range**

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It is the maximum stepping rate at which the stepper motor can operate in slewing mode developing a specified torque without losing synchronism.

### **Pull in rate**

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It is the maximum stepping rate at which the stepper motor will start or stop without losing synchronism against a given load torque  $T$ .

### **Pull out rate**

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It is the maximum stepping rate at which the stepper motor will slew, without missing steps, against load torque  $T$ .

### **Synchronism**

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This term means one to one correspondence between the number of pulses applied to the stepper motor and the number of steps through which the motor has actually moved.



### Slewing

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A stepping motor has the extraordinary ability to operate at very high stepping rates upto 20,000 steps per second. When the pulse rate is high, the shaft rotation seems rotating continuously. The operation of stepper motor at high speed is called slewing.

### Holding Torque

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It is the maximum load torque which can be applied to the shaft of the energized stepper motor without causing continuous rotation. If the holding torque is exceeded, the motor suddenly slips from the present equilibrium position and it will move one full step when the windings are energized.

### Detent torque

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It is the maximum load torque which the un-energized stepper motor can withstand without causing rotation. Detent torque is due to magnetism, and is therefore available only in permanent magnet and hybrid stepper motor. It is about 5-10 % of holding torque. Under this torque, even though the excitation increases, the rotor comes back to normal rest position. This position is the *detent position*.

## 7.14 Control of Stepper Motors

Stepper motors are commonly used for position control applications. For that purpose, the rotation of the stepper motor should be controlled precisely. The maximum step rate allowed for the required load torque can be determined from the pull-out characteristic of the motor. The stepper motor position is controlled by generating a train of pulsed by a pulse generator which is converted into the correct sequence of winding excitations by a translator. Control of step motors can be achieved in two ways: open loop and closed loop.

### 7.14.1 Open loop Control

The open loop control is simple and more widely used control scheme for stepper motor. A block diagram of a typical open-loop-stepper drive system is shown in Figure 7.17. When the command is given to the pulse generator, it generates a train of pulse which corresponds to the the number of steps for rotation and direction of rotation.

The position and direction pulse train is given to the translator. The Translator is a simple logical controller which generates the correct sequence of winding excitations to the different phases. The amplifier is provided in order to amplify this signal and

drives current in the corresponding winding. The direction of rotation can also be reversed by sending corresponding direction pulse train to the translator. Power controller consists of power electronic switching devices which receives signals from the amplifier. After receiving a directional pulse, the step motor will be able to change the direction of rotation.

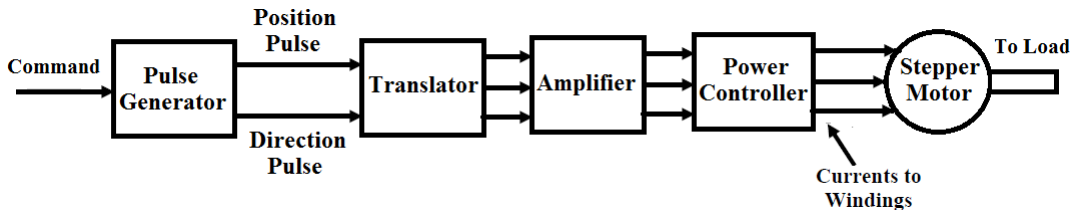


Figure 7.17: Open loop control of Stepper Motor

The major disadvantage of the open loop scheme is that it cannot detect and automatically correct the switching sequence when there is any step missing. This disadvantage can be eliminated by using closed loop control scheme of stepper motor.

### 7.14.2 Closed loop Control

The block diagram representation of a typical closed-loop-stepper motor control system is shown in Figure 7.18. Unlike open loop system, there is a feedback connection in closed loop systems which facilitates automatic correction, if there is any missed step. A pulse is sent to the driving circuit only after ensuring that the motor has rotated in the proper direction by the previous pulse. The feedback mechanism will detect the rotation in every step and the information is fed back to the digital comparator or a computer via incremental encoder. The incremental encoder is a digital transducer which is used for measuring the angular displacement of the motor shaft.

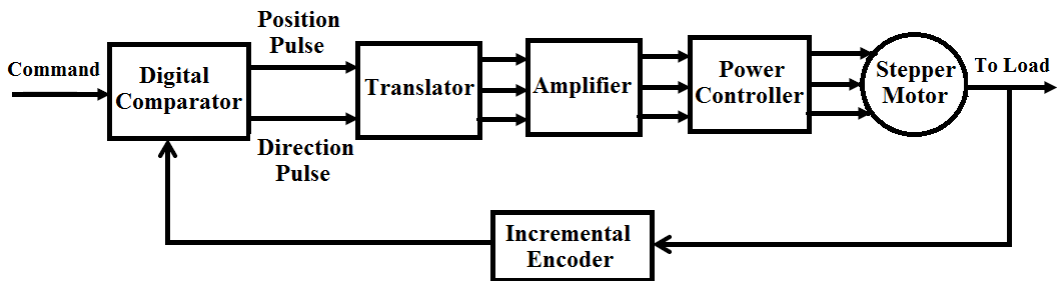


Figure 7.18: Closed loop control of Stepper Motor

The output of the incremental encoder is fed to a digital comparator or a digital computer which compares the number of steps the motor is required to move through (command) with the actual steps completed. The output of the comparator is connected to the translator which generates the correct sequence of winding excitations to the different phases.

### 7.15 Applications of Stepper Motor

Computer-controlled stepper motors are one of the most versatile forms of positioning systems. They can be used as part of an open loop system which are simpler and more rugged than closed loop servo systems. Other applications are given below.

#### **Automobile applications**

1. Air control valves
2. Throttle body motors
3. Idle air control valves
4. wiper motors

#### **Industrial applications**

1. Process control.
2. Machines Tools.
3. Pick and drop equipment
4. CNC machines
5. Packaging machinery

#### **Medical applications**

1. Scanners
2. Digital dental photography,
3. Fluid pumps
4. Blood analysis machinery.
5. Respiratory machines

#### **Office equipments**

1. Fax
2. Xerox machine
3. Printer
4. Scanners

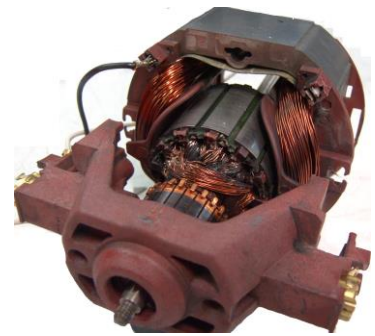
#### **Electronics**

1. Camera
2. Robotics
3. Computer peripherals
4. Graph plotters

### 7.16 Universal Motor

Universal motor is a specially designed motor which can run on either DC or single phase AC supply at approximately the same speed. Universal motor is a commutator type motor. It is a series-wound motor similar to DC series motor where the stator field windings are connected in series with the rotor (armature) windings. If the polarity of the line terminals of a universal motor is reversed, the motor will continue to run in the same direction. So it develops unidirectional torque on both AC and DC.

It is often referred to as an AC series motor. Similar to DC series motor, it has a high starting torque and a variable speed characteristics. When operating from an AC supply, the series motor develops less torque. By interchanging connections of the fields winding with respect to the armature, the direction of rotation can be changed. This motor runs at dangerously high speed without a load, and due to this, they are usually built into the device they are meant to drive. They run at lower speed on AC supply than they run on DC supply of same voltage, due to the reactance voltage drop which is present in AC and not in DC.



The Universal motor is used for the applications where high values of the speed with good speed control are necessary. This motor is used on household appliances such as vacuum cleaners, hair dryers, blenders, food mixers, drills, washing machine, sewing machines etc.

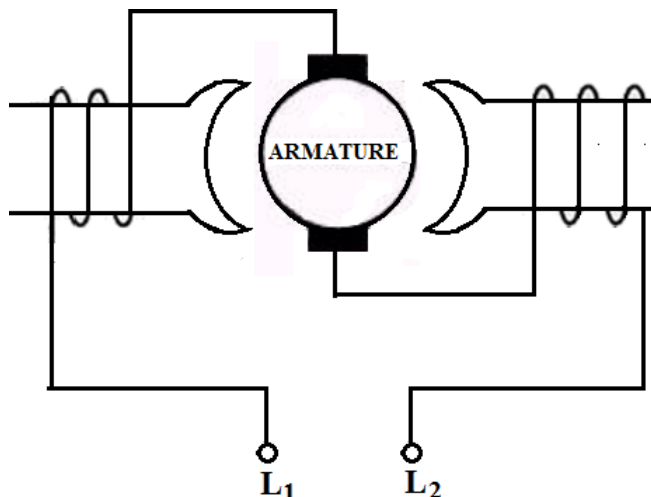


Figure 7.19: Universal Motor

### 7.16.1 Construction

Construction of a universal motor is very similar to the construction of a DC series motor. There are two types of Universal Motor:

- (i) Non-compensated Type with Concentrated Field.
- (ii) Compensated Type with Distributed Field.

Figure 7.19 shows a non-compensated universal motor. It has two salient poles and the poles are laminated. It has a concentrated field winding. The stator is laminated in order to reduce eddy current loss due to alternating flux, when motor is operated from AC supply. It has a wound type armature with laminated core having either straight or skewed slots. The armature winding is connected to the commutator.

In order to improve commutation high resistance carbon brushes are used. The compensated type universal motor consists of distributed field winding. The construction of stator core is similar to that of split-phase motor. It has a wound armature similar to that of a DC motor. Similar to split phase motor which has an auxiliary winding in addition to main winding, compensated type universal motor consists of an additional winding called compensating winding. The compensating winding is provided in order to reduce the reactance voltage which is caused due to alternating flux, when the motor runs with the AC supply. The compensated type motors are commonly used for applications which require large power output at high speed.

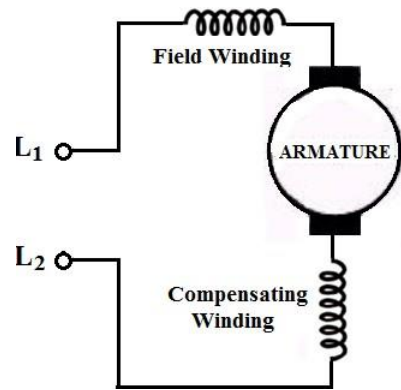


Figure 7.20: •

### 7.16.2 Principle of Operation

The principle of operation of universal motor is same as a DC motor. When a current carrying armature conductor is placed in a magnetic field, it experiences a mechanical force due to which the armature rotates. This is true regardless of whether the current is alternating or direct. Universal motors develop unidirectional torque both on DC or AC supply. When it is connected to a DC supply, it works as a DC series motor. When current flows in the field winding, it produces a magnetic field. Since the field windings and armature winding are connected in series, the same current will flow through the armature and we will get a unidirectional torque.

If the supply is AC, the polarity changes between +ve and -ve. Figure 7.21 shows the production of unidirectional torque, when the motor connected to an AC supply.

During positive half cycle, the terminal  $L_1$  becomes positive and  $L_2$  becomes negative as shown in Figure 7.21(a). The direction of current through the armature and field

windings are same. During negative half cycle, the terminal  $L_1$  becomes -ve and  $L_2$

becomes +ve. In that case too the direction of current through the armature and field windings are same as shown in Figure 7.21(b). Since the current through the armature and the field windings are in the same direction on both half cycles, the resultant torque would be an unidirectional torque.

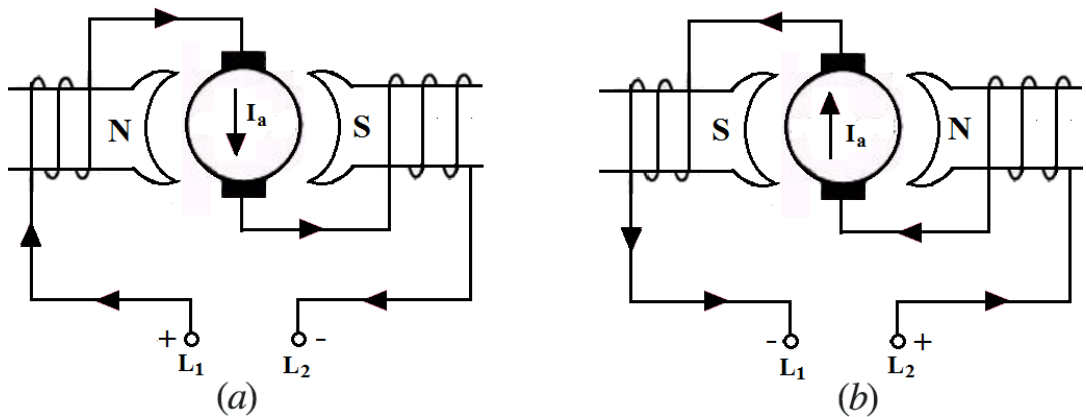


Figure 7.21: Two Pole Universal Motor

**Note**

If an ordinary DC series motor is connected to an AC supply, the following problems will occur

- ☞ Excessive eddy current loss at yoke and field cores
- ☞ Sparking at brushes
- ☞ Motor will operate at loss power factor

However, by proper modification of series motor, it can be designed as a universal motor. The modifications to be done are given below

- ☞ The field core is made up of the material having a low hysteresis loss. It is laminated to reduce the eddy current loss.
- ☞ Reactive voltage drop are reduced by reducing the area of the field poles.

**7.16.3 Advantages and disadvantages**

Advantages of universal motors are

1. It can run on AC or DC supply

2. It Is a cheap motor
3. It has good torque at low speeds
4. High starting torque
5. High speed operation with speed control is possible

Disadvantages of universal motors are

1. Noisy operation at high speed
2. Gear mechanism is required for portable tool operations
3. Regular maintenance is required for commutator and brushes
4. Brushes and commutator wear out, create sparking which can cause electro-magnetic interference

### **7.16.4 Applications of Universal Motor**

Universal motors are used in

1. Electric saw and drill
2. Vacuum cleaners
3. Blenders and food mixers
4. Washing machines
5. Sewing machines
6. Centrifugal blowers
7. Locomotives
8. Hair dryers

### **7.16.5 Torque-Speed Characteristics**

Torque-Speed characteristics of a universal motor is similar to that of DC series motor. The speed of a universal motor is low at full load and very high at no load.



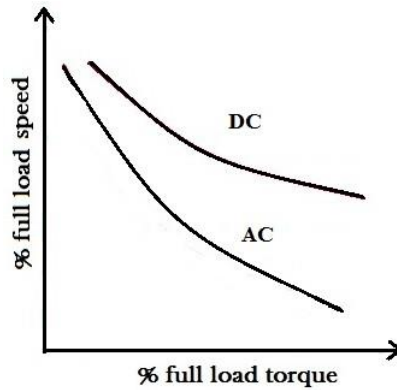


Figure 7.22: Torque-Speed Characteristics of Universal Motor

**Table 7.7: Comparison of VR stepper motor and PM stepper motor**

	<b>VR Stepper Motor</b>	<b>PM Stepper Motor</b>
1	Permanent magnet is absent in rotor	Rotor has permanent magnet
2	High torque to inertia ratio	Low torque to inertia ratio
3	Fast dynamic response	Slow dynamic response
4	High rate of acceleration	Slow acceleration
5	Small step angle is possible	Step angle is large
6	Detent torque is absent	Detent torque present
7	Salient pole rotor	Smooth cylindrical rotor
8	Large step rate	Small step rate

### EXERCISE

1. What is a stepper motor. Compare it with conventional motors ?
2. How do you classify stepper motors ?
3. Explain the principle of operation of stepper motor ?
4. What are the advantages, disadvantages and applications of step motor ?
5. Define the terms step angle and resolution.

6. Explain the working of a variable reluctance stepper motor ?
7. Explain the construction and working of a multi-stack variable reluctance stepper motor ?
8. What are the advantages and disadvantages of a variable reluctance stepper motor ?
9. Describe various modes of operation of a variable reluctance stepper motor ?
10. What is meant by microstepping in stepper motors. What are its advantages ?
11. Compare variable reluctance, permanent magnet and hybrid stepper motors ?
12. Define holding torque and detent torque of a stepper motor ?
13. Explain the open loop and closed loop control of stepper motor ?
14. Explain the static and dynamic characteristics of a stepper motor ?
15. Draw and explain the torque-speed characteristics of a stepper motor ?
16. Explain start-stop and slewing modes of operation of stepper motor ?
17. Explain the construction and working of a linear stepper motor ?
18. What are the applications of linear stepper motor ?
19. If a stepper motor has 8 stator poles and having 20 stator teeth. If rotor has 16 teeth, calculate the stepping angle and resolution ?
20. What is resonance in stepper motor ?
21. Explain pull-in torque and pull-out torque of a stepper motor ?
22. What is an universal motor ?
23. What are the different types of universal motors and their construction ?
24. What are the applications of an universal motor ?
25. What are the advantages and disadvantages of an universal motor ?
26. Explain the operation of universal motor on AC supply. Also describe how does an universal motor develop unidirectional torque on AC supply ?

# Controllers for Automation

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Control systems are an inevitable part of human life. History of mankind shows that, from the ancient times onwards he has tried to control the world he lives with the help of his intelligence. Control system means measuring any quantity of a system and maintained or altered in accordance with our desired manner. The automatic control system has become an indispensable part of human life due to its numerous applications in industry, manufacturing processes, efficient energy utilization and advanced auto-mobile control. For example, autopilot of aircraft, radar systems, intelligent motion in industrial robots, temperature control systems, pressure control systems, Numeric control in machine tools and flow process control in industries.

In recent years there has been a rapid increase in the use of digital control in control systems due to the availability of low cost digital computers. Now Digital computers are used for achieving optimal performance. Application of computer control are seen in robotic industries, manufacturing processes, household appliances and machines such as microwave ovens and sewing machines, supervisory control and data acquisition in industries and biomedical systems. Now digital control rather than analog control of dynamic systems is mainly used in industrial applications due to the advantages found in working with digital signals rather than continuous-time signals.

## 8.1 Control System

A System consisting of interconnected components designed to achieve a desired function. In a system, if an output quantity can be varied by varying any of the input quantities, that system is a control system. The output quantity which is measured and controlled is called a *controlled variable* and the quantity which is varied by a controller which can affect the controlled variable is called a *manipulated*

*variable.*

### 8.1.1 The Importance of Control Theory

The objective of control theory is to design a controller that makes a system behave in a desirable manner, that means we can predict how the system change in response to different inputs. The controller is designed with the help of mathematical models. Understanding control theory requires engineers to be well versed in basic mathematical concepts and skills, such as solving differential equations and using Laplace transform. The role of control theory is to help us gain insight on how and why feedback control systems work and how to systematically deal with various design and analysis issues. Specifically, the following issues are of both practical importance and theoretical interest:

1. Stability and stability margins of closed-loop systems.
2. How fast and smooth the error between the output and the set point converges to zero.
3. How better the control system responds to unexpected external disturbances, sensor noises, and system parameter changes.

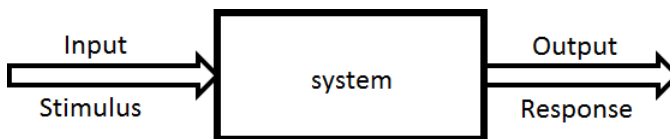


Figure 8.1: Simplified Control System

### 8.1.2 Open-Loop System

Figure 8.2 illustrates the block diagram of an open loop control system. The systems in which the output quantity has no effect upon the control of input quantity are called open loop systems. Due to the absence of feedback connection, it cannot automatically correct the variation in output. In other words, the output quantity is neither measured nor compared with the input.

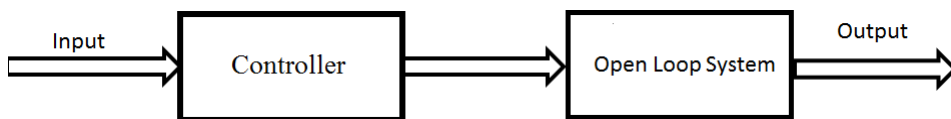


Figure 8.2: Open loop system

Output of an open loop system can be varied only by varying the input. The accuracy of system depends upon calibration. But due to external disturbances, an

## Controllers for Automation

open loop system cannot perform desired task. In the presence of external disturbances, the output can be varied only by varying the input. Systems that operates on the time set by the user are examples of open loop system. An automatic toaster is an example of open loop system as it is controlled by time. Traffic control system is another example of open loop system in which the operation is based on the time set by the user.

### Merits of open loop system

1. The open loop systems are simple and highly economical.
2. The open loop systems are more stable compared to closed loop systems.
3. Construction of open loop system is simple.
4. The maintenance of open loop systems are much easier.

### Demerits of open loop system

1. Open loop systems are inaccurate and the accuracy depends on calibration.
2. The variations due to external disturbances are not corrected automatically.
3. Open loop systems are unreliable.

### 8.1.3 Closed–Loop System

Closed loop control system is also known as *feedback control system*. Control systems in which the output (controlled variable) has an effect upon input (manipulated variable) is called a closed loop control system. Unlike open loop systems, the feedback connection in closed loop systems facilitate automatic correction of changes due to disturbances. Hence the closed loop system is also known as *automatic control system*.

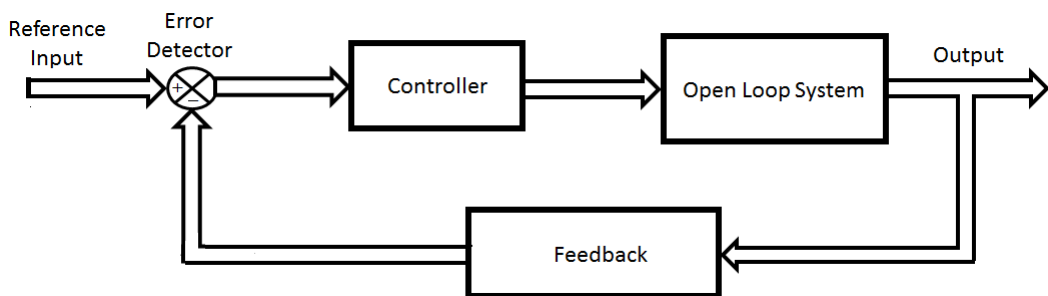


Figure 8.3: Closed loop system

Figure 8.3 shows the block diagram of a closed loop control system. The output is compared with reference input by an *error detector*. The output of the error

detector is the difference between the reference input and feedback signal. The error signal generated by the error detector is amplified and modified by a controller. The modified error signal is then fed to the plant which corrects the variation in output.

### Merits of Closed loop system

1. The closed loop systems are less affected by non linearities and distortions.
2. Accuracy of closed loop system is very high
3. The closed loop systems are accurate due to the presence of feedback because the controller modifies the actuating signal such that the error in the system will be zero.
4. The variations due to external disturbances can be corrected automatically.
5. Higher Band width

### Demerits of Closed loop system

1. Closed loop systems are more complex compared to open loop system.
2. The variations due to external disturbances are not corrected automatically.
3. Overall gain decreases due to the presence of feedback.

## 8.2 Laplace Transform

Laplace transformation is a technique for solving differential equations. The Laplace Transform converts a differential equation from the time-domain into the so-called “s-domain”, or the “*Laplace domain*.”. Differentiation becomes an algebraic operation in the s-domain. In Laplace Transform method, differential equation in time domain is first transformed to frequency domain form. After solving the algebraic equation in frequency domain, the result obtained is finally transformed to time domain form by means of inverse Laplace transform to achieve the ultimate solution of the differential equation. In other words it can be said that the Laplace transformation is nothing but a shortcut method of solving differential equation.

The Laplace transform exists for linear differential equations for which the transformation integral converges. Therefore, a function  $f(t)$  is transformable, it is sufficient that

$$\int_0^{\infty} f(t)e^{-\sigma_1 t} dt < \infty \quad (8.1)$$

For some real positive  $\sigma_1$ . If the magnitude of  $f(t)$  is  $|f(t)| < Me^{at}$  for all positive  $t$ , the integral will converge for  $\sigma_1 > a$ , The region of convergence is therefore given by

$\sigma > \sigma_1 > a$  and  $\sigma_1$  is known as the abscissa of absolute convergence. The signals

that are physically realizable always have Laplace transform.

The Laplace transformation for a function  $f(t)$  is

$$L\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st} dt \quad (8.2)$$

The inverse Laplace transform is written as

$$f(t) = \frac{1}{2\pi j} \int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)e^{st} ds \quad (8.3)$$

### 8.3 Transfer Function

The transfer function of a linear time-invariant system is defined as the ratio of Laplace transform of output to the Laplace transform of input with zero initial conditions. The concept of transfer function is applicable only for linear time, invariant, differential equation systems. If the transfer function of a system is known, the response of the system can be easily predicted for various input signals.

$$\text{Transfer Function} = \frac{\text{Laplace Transform of Output}}{\text{Laplace Transform of Input}} \quad \text{with zero initial conditions}$$



Figure 8.4: Transfer Function

Figure 8.4 shows an open loop system  $G(S)$ . The input and output of the system are represented by  $R(S)$  and  $C(S)$  respectively. The transfer function of the system with zero initial condition is given as

$$G(S) = \frac{C(S)}{R(S)} \quad (8.4)$$

#### Example 8. 1

Obtain the transfer function of the time-invariant system defined by the following differential equation with zero initial conditions.  $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 6y = x(t)$



We have,

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 6y = x(t)$$

Taking Laplace transform on both sides,

$$s^2Y(s) + 4sY(s) + 6Y(s) = X(s)$$

$$[s^2 + 4s + 6]Y(s) = X(s)$$

Where  $Y(s)$  is the Laplace transform of output and  $X(s)$  is the Laplace transform of input. The transfer function  $G(s)$  is given by,

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s^2 + 4s + 6)}$$

### 8.3.1 Advantages of Transfer Function approach

- (i) If transfer function of the system is known, the output response for any type of reference input can be predicted.
- (ii) It helps to determine the poles, zeros and the characteristic equation of a system.
- (iii) It helps in the stability analysis of the system using the location of poles.
- (iv) The differential equation governing the system can be easily obtained from the transfer function.

### 8.3.2 Disadvantages of Transfer Function approach

- (i) Only applicable for linear time invariant system.
- (ii) Initial conditions are considered as zero.
- (iii) It does not providing any information about the overall structure of a system.

## 8.4 Order of a System

The relationship between input and output of a linear, time-invariant system can be expressed by using a differential equation. The order of a control system is the order of the differential equation governing that system. If the system dynamics can be represented by an  $n^{\text{th}}$  order differential equation, then the system is an  $n^{\text{th}}$  order system.

The order of a system can also be obtained from the transfer function. The order of the system is the maximum power of  $s$  in the denominator polynomial of the transfer function. The value of  $n$  shows the number of poles of a transfer function.

## Controllers for Automation

Consider the time-invariant system represented by a differential equation,

$$a_0 \frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_n y = b_0 \frac{d^m x}{dt^m} + b_1 \frac{d^{m-1} x}{dt^{m-1}} + \dots + b_m x \quad (n \geq m) \quad (8.5)$$

Where  $y$  is the output of the system and  $x$  is the input. The transfer function can be represented as

$$\text{Transfer Function} = G(s) = \frac{L[\text{Output}]}{L[\text{Input}]} \quad (8.6)$$

$$= \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{a_0 s^n + a_1 s^{n-1} + \dots + a_n} \quad (8.7)$$

The order of the system is the maximum power of  $s$  in the denominator polynomial. Here the order of the system is  $n$ .

If  $n=0$ , the system is a zero order system

If  $n=1$ , the system is a first order system

If  $n=2$ , the system is a second order system

Consider the system represented by a transfer function,

$$G(s) = \frac{20}{s^2 + 2s} \quad (8.8)$$

Here the maximum power of  $s$  in denominator polynomial is 2. So  $G(s)$  represented a second order system.

### 8.5 Type Number of a System

The number of poles of the transfer function at the origin decides the type number of the system. If there are  $N$  number of poles at the origin then the type number of the system is  $N$ .

The transfer function can be represented as

$$G(s) = K \frac{(s + z_1)(s + z_2)(s + z_3)\dots}{s^N (s + p_1)(s + p_2)(s + p_3)\dots}$$

The value of  $N$  in the denominator gives the type number of the system

Where  $z_1, z_2, z_3$ , etc are the zeros of the transfer function

$p_1, p_2, p_3$ , etc are the poles of the transfer

function  $N =$  Number of poles at the origin

(8.9)

Consider the transfer function given below

$$G(s) = \frac{20}{s(s+2)} \quad (8.10)$$

$$G(s) = \frac{20}{(s+2)} \quad (8.11)$$

Transfer function in equation 8.10 represents a type 1 transfer function and equation 8.11 represents type 0 transfer function.

## 8.6 Time Response

The time domain response of a system is the output of the system as a function of time. The time domain analysis of a system involves the transient response analysis and steady state analysis. Transient analysis of the system is the study of the output of a system when it goes from initial state to final state. Steady state analysis is the study of the system output when the time approaches to infinity. These two analysis are generally involved in the design of feed back control signal for specified input signal. The time response of the system is obtained by solving the differential equation governing that system. It can also be obtained from the transfer function of the system. So the output can be written as,

$$c(t) = c_{tr}(t) + c_{ss}(t)$$

where  $c(t)$  is the system response,  $c_{tr}(t)$  is the transient response and  $c_{ss}(t)$  is the steady state response.

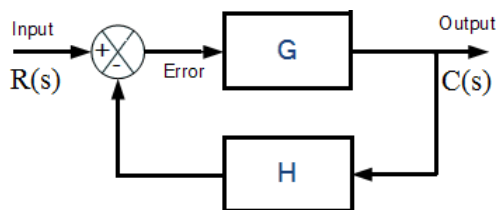


Figure 8.5: Closed Loop System

Consider a closed loop system as shown in Figure 8.5.  $C(s)$  is the output of the system in Laplace domain and  $R(s)$  is the input in Laplace domain.  $G(s)$  is the open loop transfer function.

$$\text{The closed loop transfer function, } T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad (8.12)$$

$$\text{The response in s-domain, } C(s) = R(s) \frac{G(s)}{1 + G(s)H(s)} \quad (8.13)$$

For a given input signal, the inverse Laplace transform gives the time response of the system,

$$c(t) = L^{-1} [C(s)] = L^{-1} R(s) \frac{G(s)}{1 + G(s)H(s)} \quad (8.14)$$

### 8.7 Test signals

Test input signals are used to verify the design, both analytically and by experiments. Engineers usually use standard test signals to analyse a system's performance. Test signals can be easily generated in laboratories as they are simple functions of time. The commonly used test signals are step signal, ramp signal, parabolic signal, impulse signal and sinusoidal signal. The performance of a system can be evaluated with respect to these test signals.

#### 1. Step Signal

Step input represents a constant command such as position, velocity or acceleration. It is being used to see the transient response of system as it gives you the idea about how the system reply to interruption and somehow the system stability.

**eg** :- sudden rotation of a shaft, switching on a constant voltage in an electrical circuit, sudden opening or closing a valve.

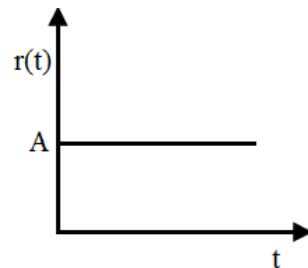
$$r(t) = A u(t)$$

Where A is a constant

$$u(t) = \begin{cases} 1, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases}$$

For unit step function  $A=1$

$$r(t) = u(t) \quad (8.15)$$



#### 2. Ramp Signal

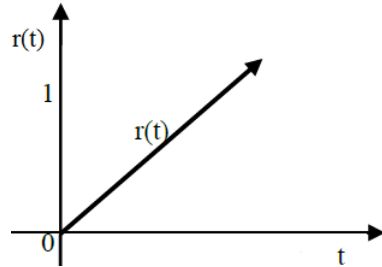
It is constant rate of change in input that is gradual application of input. Ramp Function  $r(t)$  states that the signal will start from time zero and instantly will take a slant shape and depending upon given time characteristics (i.e. either positive or negative) the signal will follow the straight slant path either towards right or left

**eg**:- Altitude Control of a Missile

$$r(t) = \begin{cases} At, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases} \quad (8.16)$$

For unit ramp signal  $A=1$

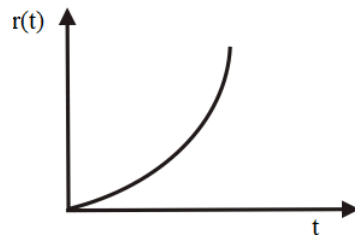
$$r(t) = t$$



### 3. Parabolic Signal

The parabolic signal is commonly used as a test signal for constant acceleration input. In a parabolic signal, the parabolic signal varies as square of the time from  $t=0$ . Parabolic signal is faster than ramp signal.

$$r(t) = \begin{cases} At^2, & \text{if } t \geq 0 \\ 0, & \text{if } t < 0 \end{cases} \quad (8.17)$$

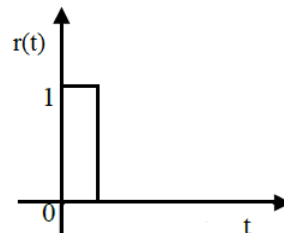


### 4. Impulse Signal

Impulse response in control system imitates sudden shock quality of actual input signal. Impulse is the output of system when given by small input. Impulse response emphasis on change in the system in reaction to some external change. It is the reply of the system to the direct delta input.

The unit-impulse function  $\delta(t-t_0)$  can be obtained by taking the derivative of unit-step function.

$$r(t) = \begin{cases} 0, & \text{if } t \neq 0 \\ \lim_{t \rightarrow 0} \delta(t)dt, & \text{if } t > 0 \end{cases} \quad (8.18)$$



The impulse function whose area is equal to unity is called the unit-impulse function or the Dirac delta function. The unit impulse function occurring at  $t = t_0$  is usually denoted by  $\delta(t - t_0)$ . Lightning is an example for impulse.

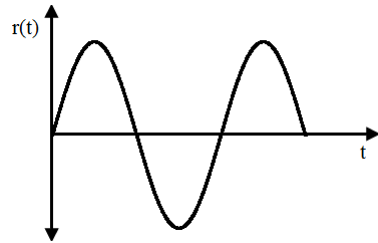
### 5. Sinusoidal Signal

Sinusoidal waves represent pure AC current and voltage. It is a periodic signal with positive and negative half cycles.

Sinusoidal signal is commonly used for frequency response analysis.

$$r(t) = \begin{cases} 0, & \text{if } t < 0 \\ A \sin \omega t, & \text{if } t \geq 0 \end{cases} \quad (8.19)$$

Where A is the amplitude and  $\omega$  is the frequency in *rad/sec*



### 8.8 First Order System

If the differential equation governing the system is a first order differential equation, then the system is called the first order system. The order of the differential equation is the highest degree of derivative present in an equation. The standard form of closed loop transfer function of a first order system is given by

$$\frac{C(s)}{R(s)} = \frac{1}{1 + \tau s} \quad (8.20)$$

Where  $\tau$  is the time constant of the system

### 8.9 Second Order System

If the differential equation governing the system is a second order differential equation, then the system is called second order system. The standard form of closed loop transfer function of a second order system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (8.21)$$

Where  $\omega_n$  is the natural frequency of oscillation

$\zeta$  is the damping ratio of the system

The characteristic equation of the second order system is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \quad (8.22)$$

The roots of the characteristic equation is

$$\begin{aligned}
 s_1, s_2 &= \frac{-2\zeta\omega \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} \\
 &= \frac{-2\zeta\omega \pm \sqrt{4\omega_n^2(\zeta^2 - 1)}}{2} \\
 &= \frac{-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}}{1} \\
 &= -\zeta\omega_n \pm \omega_n\sqrt{(-1)(1 - \zeta^2)} \\
 &= -\zeta\omega_n \pm j\omega_d(1 - \zeta^2)
 \end{aligned} \tag{8.23}$$

Where  $\omega_d = \omega_n \sqrt{1 - \zeta^2}$  is the damped frequency of oscillation of the system in rad/sec

The transient response of the second order system is given by the general solution

$$x(t) = Ae^{s_1t} + Be^{s_2t} \tag{8.24}$$

- Case 1** :  $\zeta > 1$ , The roots are real, negative and unequal (over damped System)
- Case 2** :  $0 < \zeta < 1$ , complex conjugate roots with negative real parts (under damped system)
- Case 3** :  $\zeta = 1$ , real, negative and equal roots (critically damped system)
- Case 4** :  $\zeta = 0$ , pure imaginary roots (undamped system)
- Case 5** :  $\zeta < 0$ , complex roots with positive real parts (Unstable system)

**Natural frequency of oscillation,  $\omega_n$**

It is the frequency of oscillation of the system without damping. For example the frequency of oscillation of series RLC circuit at resonance(resistance shorted) is the natural frequency of oscillation.

**Damping Ratio**

The damping ratio is defined as the ratio of actual damping to critical damping. The response  $c(t)$  of second order system depends on the value of damping ratio. Based on the value of  $\zeta$  the system can be classified as



1. Undamped system ( $\zeta = 0$ )
2. Under damped system ( $0 < \zeta < 1$ )
3. Critically damped system ( $\zeta = 1$ )
4. Over damped system ( $\zeta > 1$ )

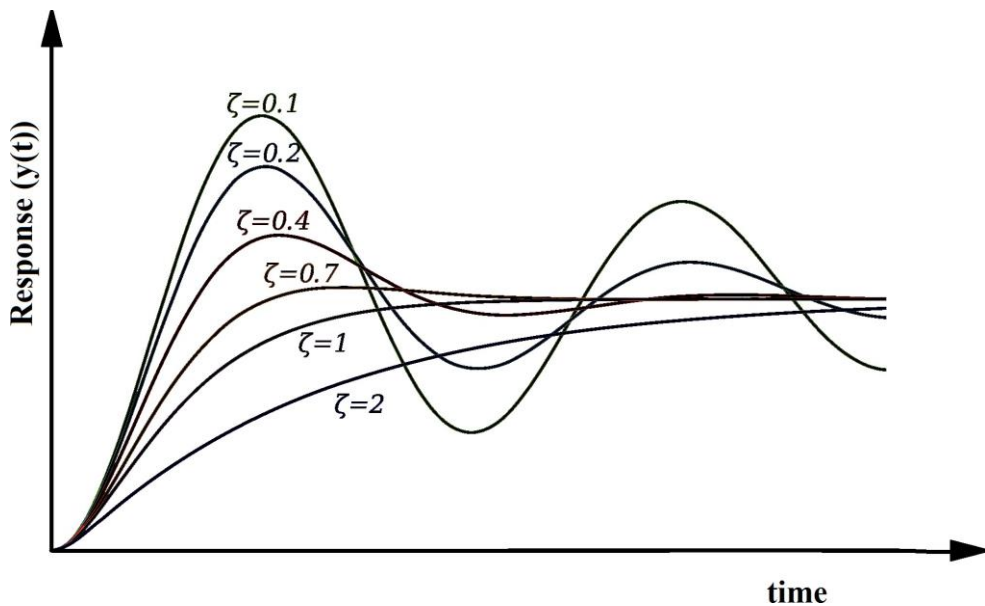


Figure 8.6: Output response for different damping ratio

### 8.10 Controllers for Automation

The word 'Automation' is derived from greek words "Auto" which means self and "Matos" which means moving. Therefore Automation is a mechanism in which the system moves itself. Automated systems also achieve significantly superior performance than manual systems. Automation is a set of technologies which helps in the conversion of a work process, a procedure, or a systems to automatic without significant human intervention. In industry, automation can be used to reduce costs, improve quality, increase manufacturing speed and reduce cost of production.

If a process is automated, then it requires closed loop or feedback control. Closed loop control system requires a controller. A *controller* is one which compares controlled values with the desired values and it continuously monitors and changes a process variable to achieve the desired performance. The various types of controllers are used to improve the performance of a control system. In this section we will discuss about various controllers which are available to achieve the overall control requirements of an electromechanical system. The controllers used in electromechanical systems are:

### 8.10.1 Motion controllers

Motion controllers are the main part of any motion control system. A motion controller creates the control signal to control the motor to meet the requirements. They are able to control the speed and position of one or a number of axes, either individually or when undertaking a coordinated movement. Modern motion controllers may also provide data-management facilities, input and outputs channels, communication, input-output ports etc.

### 8.10.2 Multi-axis CNC controller

A multi-axis CNC controller contains a number of motion controllers for the axes of the machine tool. It together with a system will generate the required motion profile. It consists of microcontroller which is capable of controlling a number of motion axis simultaneously. Many multi-axis controllers are available which works on the user defined program. See section 8.15.1 for details.

### 8.10.3 Programmable logic controllers

PLC is a digital computer used for the automation of various electro-mechanical processes in industries. A programming language known as the *Ladder Logic* is used to program the PLC. It is specially designed to survive in adverse conditions and shielded from heat, cold, dust, and moisture etc.

A PLC has many input terminals, through which it interprets HIGH and LOW logical states from sensors and switches. It also has many output terminals, through which it's outputs HIGH and LOW signals to power lights, solenoids, small motors, and other devices facilitating themselves to on/off control. See section 8.16 for details.

## 8.11 Servo Control

Servo control has become an integral part of automatic control system, including in manufacturing, production, transportation, robotics, biomedical applications etc. A servo control system can be defined as a system that is able to control some variables to track the reference input or set point. Now servo control or servomechanism is commonly used in feedback control system in which the controlled variable is mechanical position or time derivative of position (velocity and acceleration). It is an automatic device that uses error-sensing negative feedback to get desired output. So, the main objective of a servo control is to maintain the output of a system at the desired value in the presence of disturbances. Servo control is commonly used in the automation of machine tools, satellite-tracking antennas, remote control and navigation in air planes etc.

A servo system is an automatic control system . When reference input signal or command signal is applied to the system, it is compared with output signal using an error detector or comparator. The output of the error detector is the difference between the reference input and the feedback signal by a feedback sensor. The error

signal generated by the error detector is amplified and modified by a controller. The modified error signal is then fed to the controlled device (motor) which corrects the variation in output.

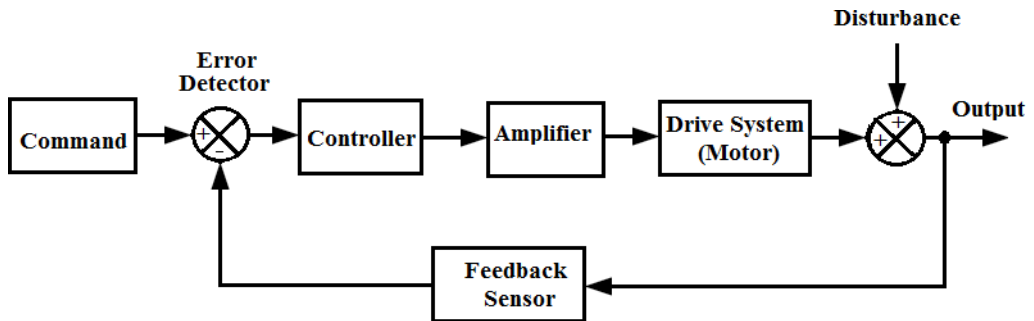


Figure 8.7: Servo control System

When the desired output is achieved, there is no logical difference between reference input signal and the output signal of the system. Hence, the servo control maintains the output of a system at the desired value even in the presence of disturbances.

### 8.12 Digital controllers

Now a days due to the advancement in digital computers and embedded system, the use of digital controllers have been introduced in control system to improve the tracking performance for complex systems. In a digital control system, the control algorithm is implemented in a digital computer or a micro controller. Digital systems has so many advantages compared to analog systems.

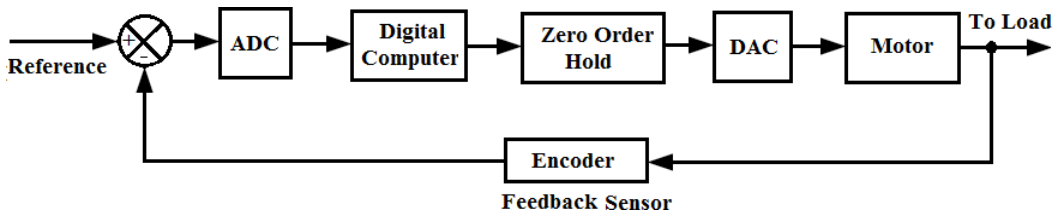


Figure 8.8: Digital Control System

When a digital controller is used within a servo controller, the data will be processed at specific intervals, in discrete samples for data acquisition and processing. The continuous time system are analysed uses Laplace transform techniques. But for the analysis of sampled-data system or digital system, the data can be transformed from the continuous s-domain to the discrete z-domain by the application of the relationship  $z = e^{sT}$ , where T is the *sampling period*.

The block diagram of a typical digital control system is shown in Figure 8.8. The analog to digital converter (ADC) and sampler is used to convert the continuous time error signal into a sequence pulses as shown in Figure 8.9 or digital signal and the digital signal is given to a digital computer. The output of the digital computer is again a discrete (digital) signal. A zero order hold (ZOH) ensures that the output is held constant between the samples. The output of the digital computer are decoded into analog signal using a digital to analog converter (DAC). The output of the DAC is applied to the motor after proper amplification. The continuous time signal then controls the motor. So the signal is in sampled form in digital computer and are continuous in other parts of the system. This kind of system is also known as *sampled-data control system*.

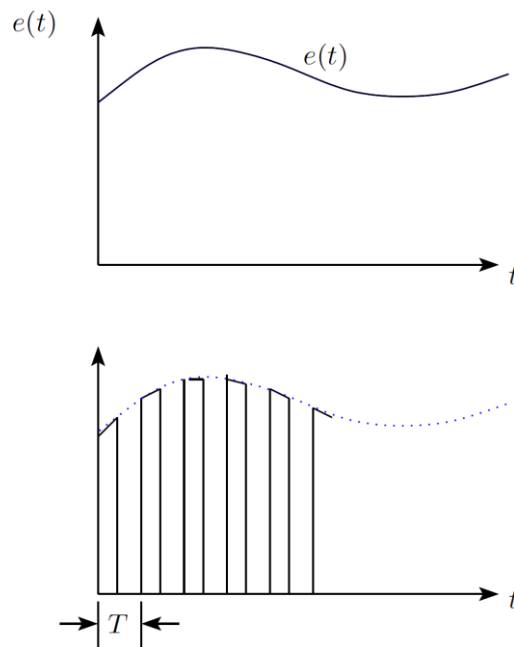


Figure 8.9: Sampled error Signal

The output is fed back to the input using a feedback sensor or an *encoder* or a resolver fitted to the load or to the motor. The position of the motor is determined using the encoder and this is compared with the reference input, to obtain the error signal. The output of the digital-to-analogue converter is changes according to the value of error signal and the motor changes its position.

### 8.13 Advanced control systems

The implementation servomechanism by means of a range of advanced techniques is now possible. Advanced control techniques includes adaptive control, artificial-neural networks, fuzzy logic etc. Adaptive control techniques are most commonly used in

servomechanism. An adaptive controller can modify its behaviour in response to the changes in the dynamics of the system and if there is any disturbance. It can also compensate the parameter variation occurring in the system. Among the various adaptive-control techniques, model reference adaptive control is the most widely used.

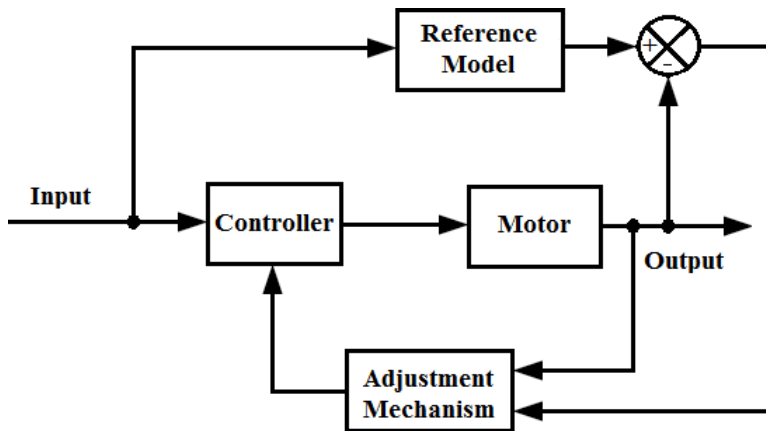


Figure 8.10: Model Reference Adaptive Control

In model reference adaptive control method, an appropriate reference model is selected and an adaptation algorithm is designed that is capable of modifying the feedback gains of the control system. The desired performance for an external command is expressed in terms of the reference model. The adaptive control algorithm modifies the errors between the output of reference model and the output of the system.

Adjustment Mechanism is used to alter the parameters of the controller so that the motor could track the reference model. The controller consists of adjustment parameters. As this is a common control strategy, complex computations are not required, and it can be easily implemented on a low-cost microprocessor. Another advantage of such approach is that the prior knowledge of complex mathematical models of the system dynamics and the characteristics of load is not required.

### 8.14 Digital signal processors

Digital Signal Processing (DSP) deals with processing on signals to get desired output/response on digital processors. Digital signal processors is a specially designed powerful microprocessor that is capable of processing data in real time. In DSP control, the signals are converted from analog to digital, manipulated digitally, and then converted again to analog form. This real-time capability makes a DSP suitable for applications such as the sensorless control of brushless motors. Digital signal processors reduces system cost and provides high speed and high resolution. Conventional motor control are done using analog components. However there are several draw-

backs with analog systems. Digital systems offer improvements over analog design. They are:

- ☞ Better accuracy
- ☞ Digital systems are independent of temperature, ageing and other external parameters.
- ☞ Control strategy can be changed easily by upgrading the software.
- ☞ Digital system can be cascaded without any loading problems.
- ☞ Higher accuracy in digital systems compared to analog systems.
- ☞ Complex control algorithms can be implemented by DSP method.
- ☞ Enables a reduction of harmonics using enhanced algorithms,

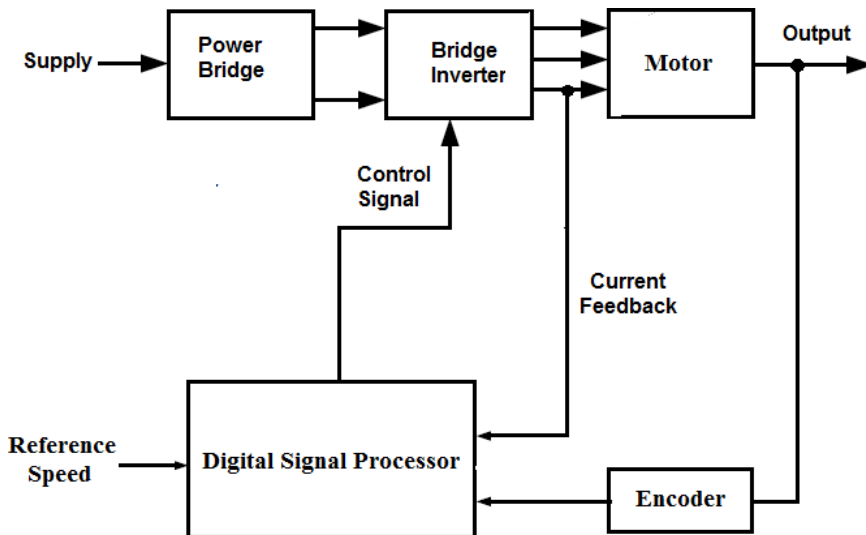


Figure 8.11: Digital Signal Processors

Analog to digital converters (ADC) is used to process the position and current data into digital format for processing in DSP. Using a DSP, efficient speed and current algorithm can be executed to control the motor. DSP receives feedback data from the position sensor, which can be either a resolver, an encoder, or a Hall effect sensor depending on the motor type. DSP also receives current feedback from one of the motor phases.

Analog to digital converters is used to process the position and current data into a digital number of a suitable format for use by DSP. Multi-channel ADCs are used for

simultaneous sampling to maintain correct phase information, and can be integrated within the DSP.

### 8.15 Motion controllers (Motor controllers)

Motion controllers are very important for any motion control system. It produces the control signal to control the motor for the generation of position, velocity, and acceleration profiles. The degree of intelligence of a motion control system mainly depends on motion controller.

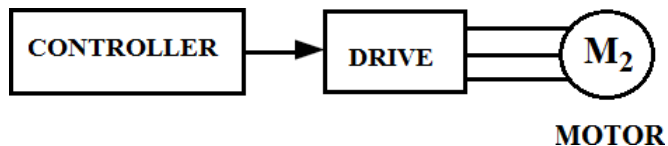


Figure 8.12: Motion Controller

The same motion controller can be used with electric servo, pneumatic servo and hydraulic servo systems. Modern motion controllers may also provide data-management facilities, input and outputs channels, communication, input-output ports etc. It also provides user interface between user and the machine. The selection of controller strategy is based on the number of axes and on the degree of coordination between the axes. The two types of motion controllers are follows:

#### 8.15.1 Axis Controllers

A multi-axis controller consists of microcontroller which is capable of controlling a number of motion axis simultaneously. Many multi-axis controllers are available which works on the user defined program. Pre-programmed or customised motion controllers are available. In that case, users have to give equation parameters and the limiting values.

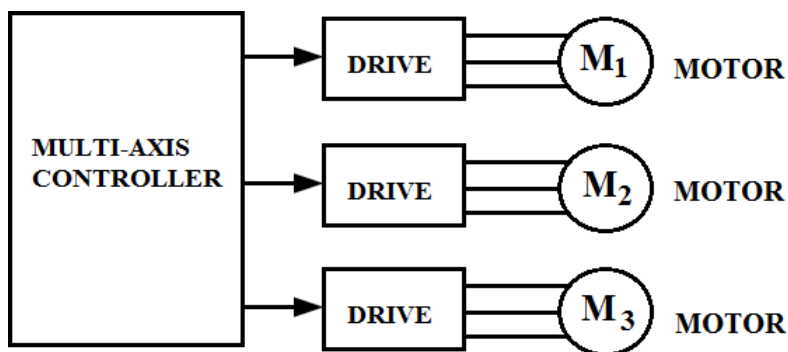


Figure 8.13: Axis Controller

Multi-axis controllers are also available as motion control cards having standard sockets which enables the user to integrate the system more easily. Cards which can



control upto eight axes are available. Using a single card, the coordinated movement between different axes can be done.

In a number of cases as shown in Figure 8.14, it is possible for one drive to act as a master unit, while the other drives are directly synchronised to maintain a zero position error between themselves and the master drive.

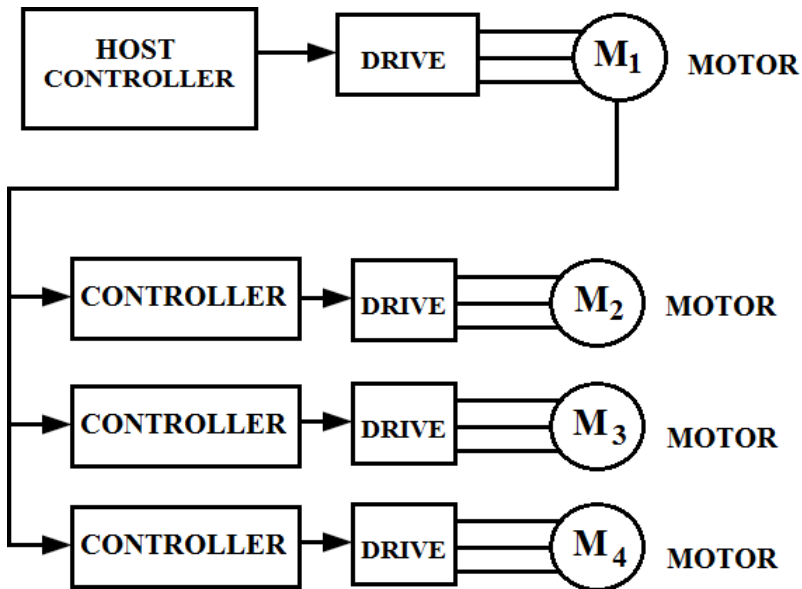


Figure 8.14: Axis Controller with Master Drive

### 8.15.2 Machine tool controllers

Machine tool controllers accept the instruction from the program and converts it to corresponding signals which controls various actions of machine tools.

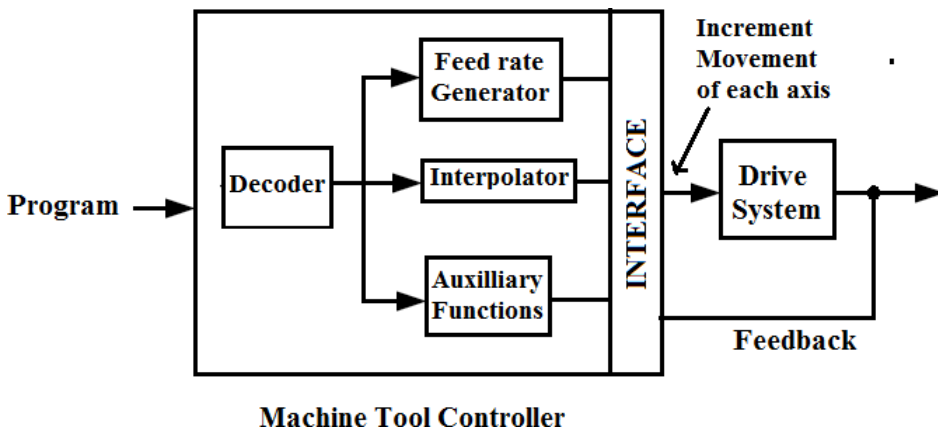


Figure 8.15: Machine Tool Controller

In a numerically controlled machine, the input instructions are usually provided by the program. The machine tool responds to the electrical signals from the controller and executes various slide motions and spindle rotations. Machine tool controllers also have a set of industry standard program command codes for machine-tool functions. These codes range from simple on-off commands to a number of canned cycles. A canned cycle is a preprogrammed subroutine program for different operations like boring, cutting, drilling etc.

The other functions of the controller are interpolation and feed rate generation. To shape a complex curve at constant feed rate, the controller gives instructions to various tools and its coordinated movement produces the required shape. Interpolation means the controller breaks the curve into small parts for each controlled motion.

### 8.16 Programmable Logic Controllers

Programmable Logic Controller (PLC) is a digital electronic device designed for multiple inputs and output operations. It is mainly used for automation in various industries. It was first used by the automotive industry to develop a system to change the control logic without the need to totally rewire the system. It has a programmable memory to store the instructions for implementing specific functions such as on/off control, logic, sequencing, timing, counting, arithmetic and data handling. The main advantage of PLC is that the user can modify the control strategy by changing the program without rewiring the input and output devices. So PLC offers a flexible control system. The main applications of PLC are on-off control, sequential control, feedback control and motion control. The PLC monitors the inputs and outputs of the system according to the program and generates control signals to control the system.

PLC was initially designed to replace relay logic boards. It is designed to minimize the number of control relays in a process. In a PLC controlled system, push-buttons, limit switches, sensors etc. can be used as input devices. Similarly, motors, valves, relays, solenoids, indicating lamps etc. can be directly wired as the output of PLC. The operator enters the program into the memory of the PLC and PLC controls the system according to the program.

#### 8.16.1 Internal Architecture

Figure 8.16 shows the block diagram representation of PLC. The CPU is the central processing unit containing the microprocessor which reads the input signal and carries out the control actions, according to the program stored in its memory. It also controls the output devices by sending control signals.

The program is stored in the program memory. The input and output modules are used to receive information from external devices and communicate information to external devices. The power supply unit is needed to convert the mains AC voltage to the low DC voltage (24 V) necessary for the processor and other interface modules.

The major blocks constituting a programmable logic controller are Power supply, CPU, Input Output system, Memory and buses.

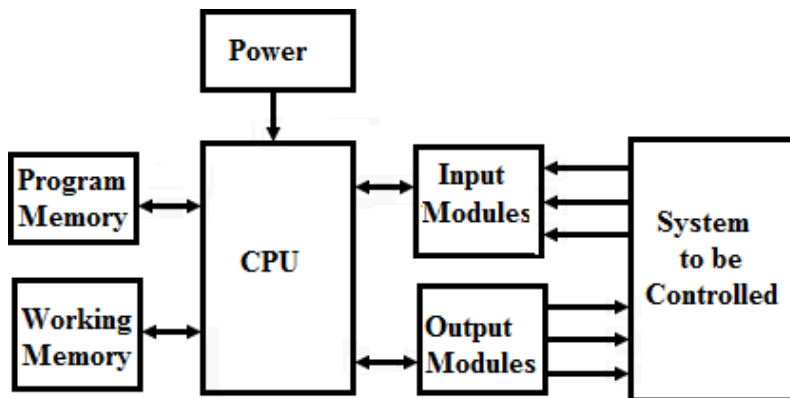


Figure 8.16: PLC Block Diagram

### CPU

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It is the brain of the CPU which depends on the microprocessor concerned. It accepts the input and carries out all the operations based on the control program. CPU consists of an arithmetic and logic unit (ALU) that is responsible for data manipulation and carrying out arithmetic operations and a control unit that is used to control the timing of operations.

### Power Supply

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It generally works on a power supply of about 24 V and used to power input and output devices. It converts the input power from 240 V AC/ 120 V AC to the rated 24 V DC.

### Memory

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All PLCs contain both RAM and ROM in varying amounts depending upon the design of the PLC. Program memory is used to store the program. A portion of RAM is used as the working memory.

### Input/Output Modules

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The input module consists of devices like sensors, switches etc. The input from the sources is connected to the PLC through the input connector rails. The output

module can be a motor or a solenoid or a lamp or a heater, whose output is controlled according to the program.

### 8.16.2 Advantages of PLC

1. Rugged in construction to withstand vibrations, temperature, humidity and noise.
2. Flexible
3. Low power consumption
4. Fast response
5. Reliable due to the absence of moving parts
6. Simple wiring required
7. Easily programmed using ladder programming
8. Easily to repair and maintain

### 8.16.3 PLC Ladder programming

Ladder programming is commonly used in PLC. It is the oldest programming language for PLC. Each ladder program consists of two vertical lines representing the *power rails*. The circuit is connected in between these vertical lines (power rails). It is considered as the *rungs* of the ladder. Each rung of the ladder program starts with an input or series of inputs and end with output. The main ladder logic symbols are shown below,

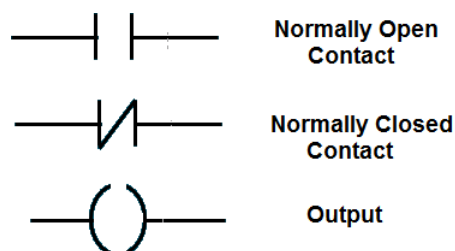


Figure 8.17: Symbols

Once the program is loaded, it is permanently stored in the PLC, so once the PLC is installed it becomes an integral part of the machine. An operator can easily reprogram the PLC and modifications to the system can be done easily.

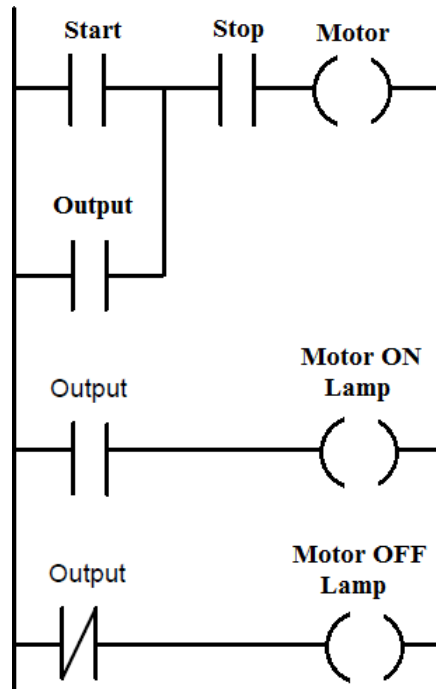


Figure 8.18: Ladder Diagram

Figure 8.18 shows the ladder diagram of a motor controlled by stop and start push button switches and for which one signal lamp must be illuminated when the motor is on and another lamp must be illuminated when motor is off. When the start switch is closed, the program will start. When stop switch is closed, output contact is energised and its contacts close. This makes the motor on and also the switching off of the motor off lamp and the switching on of the motor on lamp. To switch the motor off, motor off switch is opened. Now it results in switching on of the motor off lamp and the switching off of the motor on lamp.

### EXERCISE

1. What are control systems ?
2. Define open loop and closed loop control system ?
3. What are the advantages and disadvantages of open loop system ?
4. Compare open loop and closed loop systems ?
5. What is feedback. What are its effect on systems.

6. Explain the significance of Laplace transform in control system ?
7. Define transfer function. Explain its role in control system ?
8. What are the advantages and disadvantages of transfer function approach ?
9. Explain different testy signals ?
10. Describe different types of controllers used in automation ?
11. Explain servomechanism ?
12. Explain digital control techniques in drives ?
13. Describe various advanced control techniques used in drives ?
14. Explain multi-axis controllers ?
15. Explain machine tool controllers
16. Describe the role of DSP in drives ?
17. Explain different motion controllers ?
18. Describe the working of PLC ?

